Network throughput under dynamic user equilibrium: Queue spillback, paradox and traffic control

Frontier of the MFD study
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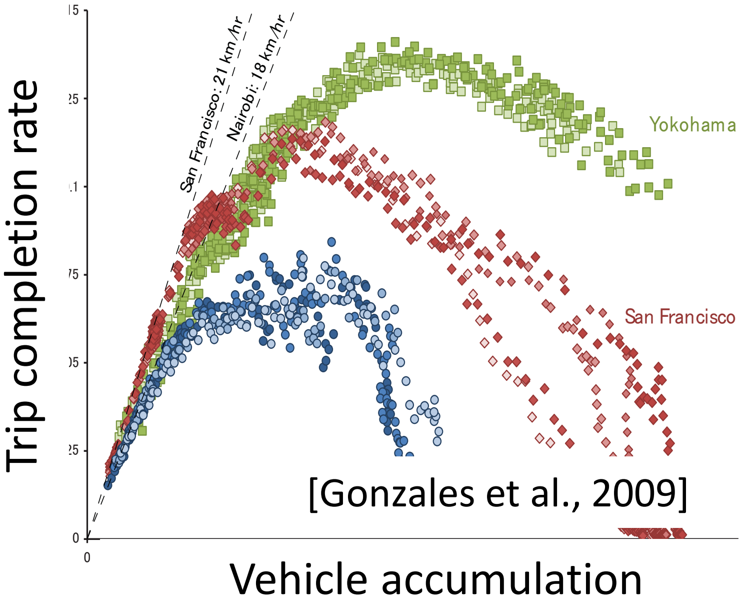
Introduction

❖ Macroscopic Fundamental Diagram (MFD) [*Daganzo* (07)]

- Accumulation vs. Network throughput (steady-state)
  - Network throughput: outflow from the network
- Useful tool for robust (without prediction) traffic control
  - monitor traffic performance via real-time accumulation

✓ Microscopic mechanisms behind macroscopic behaviors are not completely understood

- ➢ Why decreasing branch of an MFD occur ?
- ➢ What affects the shape ?

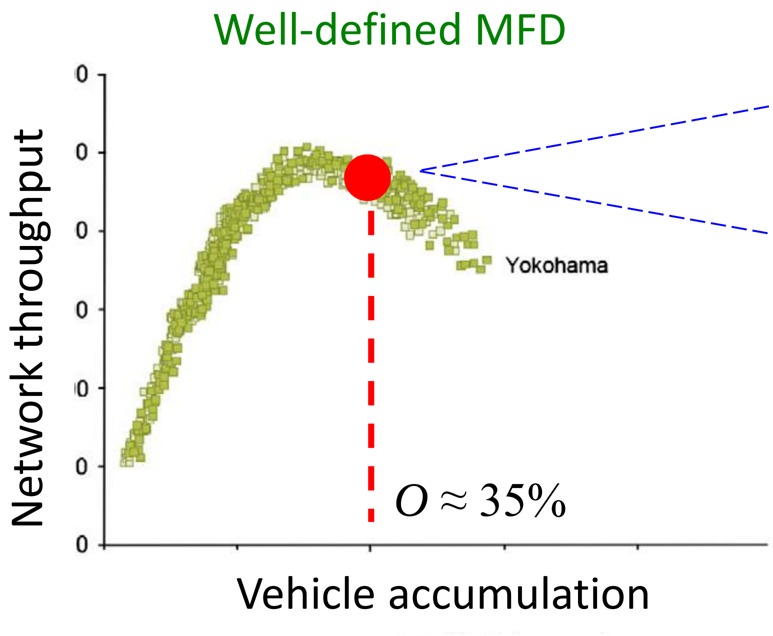


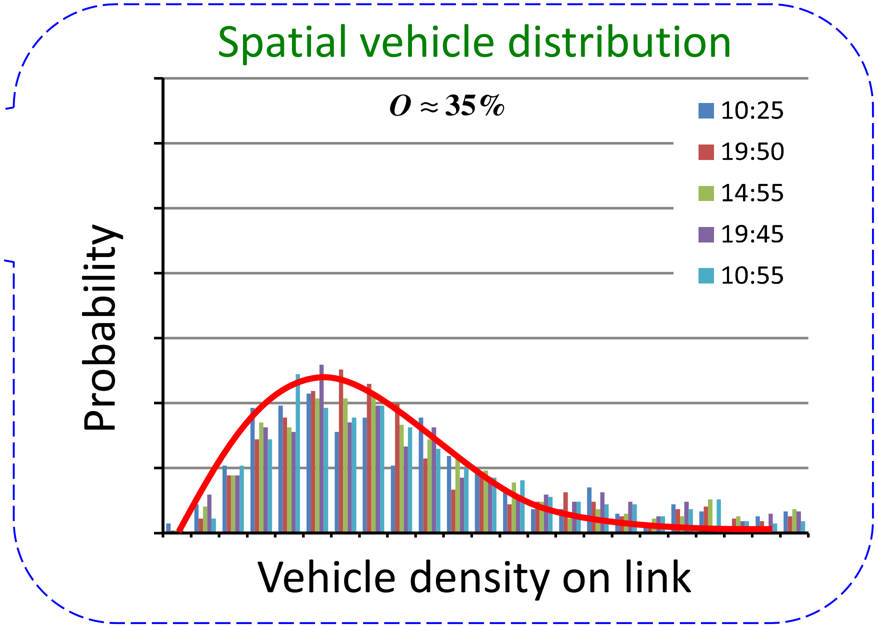
[Gonzales et al., 2009]

Spatial vehicle distribution [Geroliminis & Sun, 2011]

- Spatial vehicle distribution: If the statistically identical
  - Network throughput does not scatter for a given density
  - ➢ Characterize the network throughput

✓ Abstract spatial factors of network: difficult to clarify mechanisms

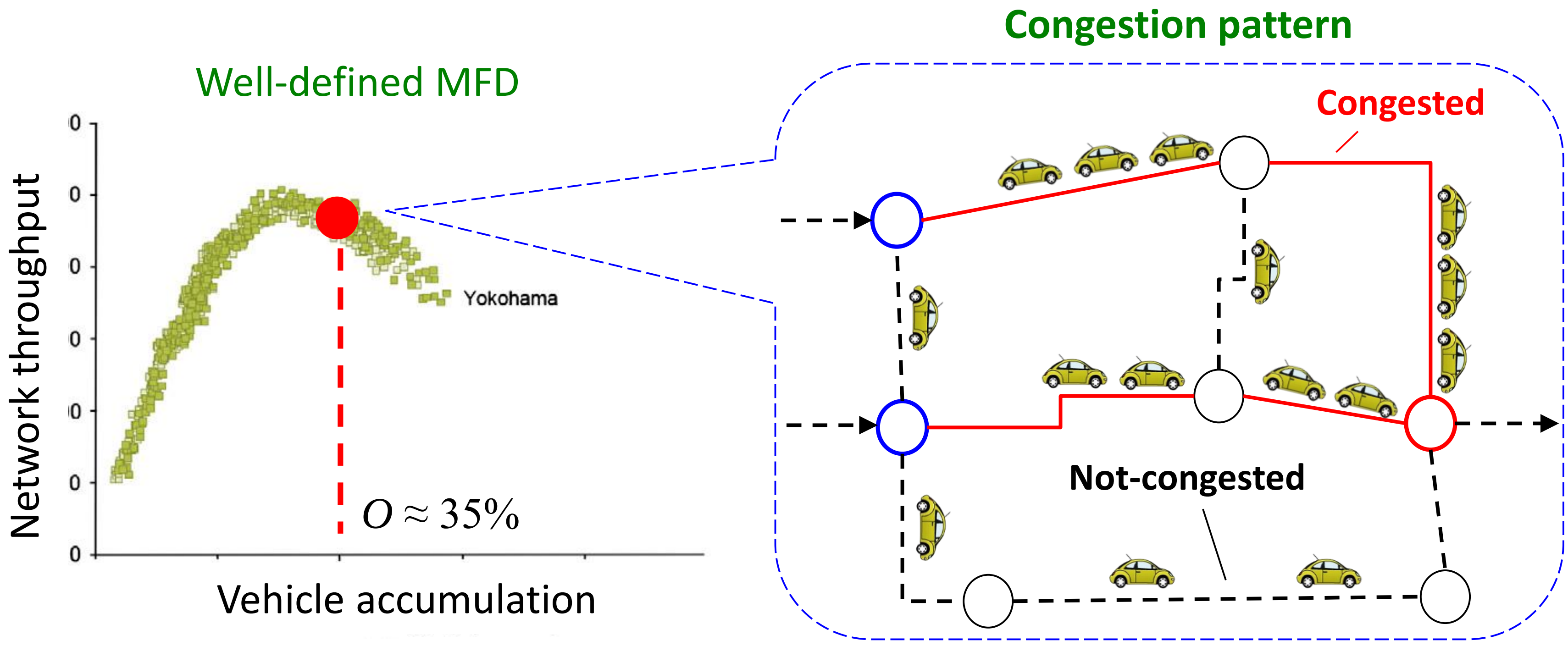




Spatial vehicle distribution

- Spatial vehicle distribution: identical for a given density
  - Suggestion: congestion pattern is reproducible

➢ We might clarify mechanisms by linking congestion patterns and MFD



Previous studies

❖ Heterogeneity of vehicle distribution

- Congestion tends to distribute unevenly due to random route choice [*Mazloumian et al. (10), Daganzo et al. (11)*]
  - Leads to a decrease in network performance

✓ Only analyze the correlation between heterogeneity of congestion and the shape of an MFD

❖ Route choice [*Leclercq and Geroliminis (13), Laval et al. (17)*]

- Induce or mitigate an uneven distribution

✓ Network structure is restrictive (a parallel route)

Far from the global understandings of the connection btw congestion pattern and the MFD

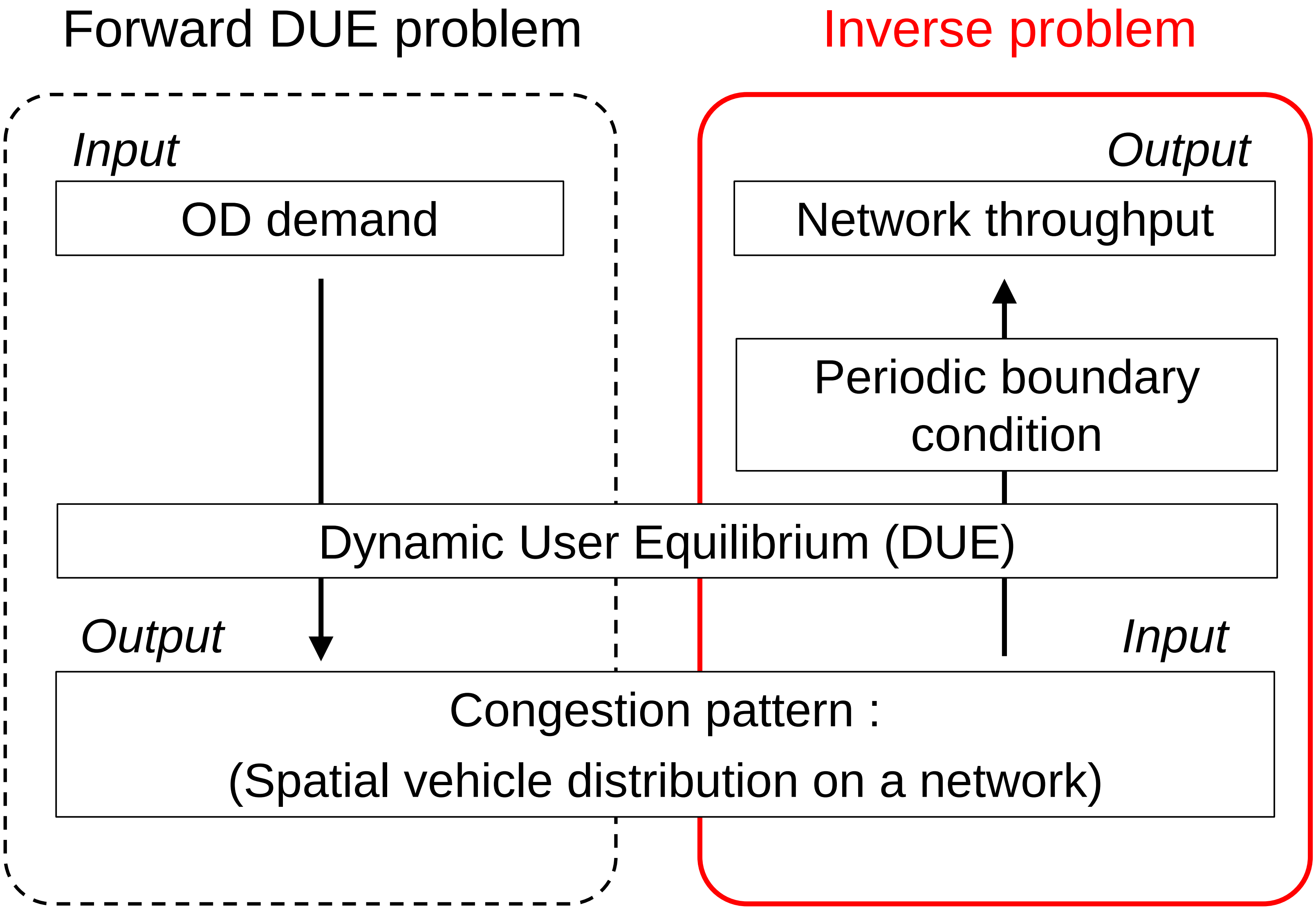
Objective

Clarify the relationship between the network performance and congestion patterns

❖ Methodology (target: one-to-many network)

- Derive network throughput analytically for a given congestion pattern
  - Solve an inverse problem of the Dynamic User Equilibrium (DUE) assignment
- Conduct sensitivity analysis to clarify the mechanisms
  - Identify queue spillback patterns decreasing throughput
  - Application: Paradox and signal control strategy

Framework of the methodology



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Dynamic User Equilibrium (DUE)

❖ Definition and Decomposition Properties [Kuwahara & Akamatsu, 93]

No user could reduce his/her travel time by changing his/her route

- Equilibrium concept with the FIFO discipline
  - The order of departure must be kept at all the node
  - FIFO btw OD pair is established

➢ DUE problem: decomposed w.r.t the departure time

❖ Equilibrium conditions

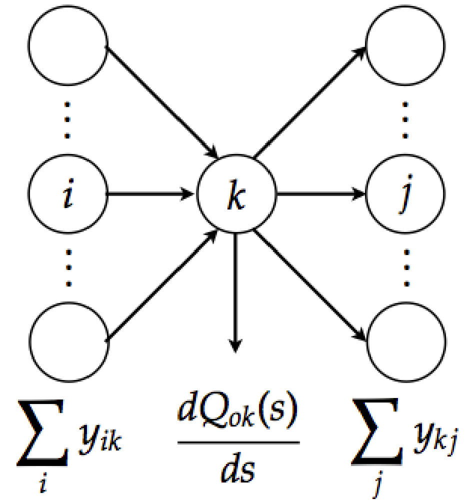
- Flow conservation
- Route choice principle
- Link travel time

(a) Flow conservation at node

❖ Flow conservation at node k

- Arrival flow rate – departure flow rate – demand = 0

$$\boxed{\sum_{i \in I(k)} y_{ik}^{s} - \sum_{j \in O(k)} y_{kj}^{s} - \frac{\mathrm{d}Q_{ok}(s)}{\mathrm{d}s} = 0}$$



- y_{ij}^{s} : Flow rate on link (*i*,*j*) w.r.t *s*
- $Q_{ok}(s)$: Cum demand with destination *d* departing from origin s until time s

(b) Route choice principle

❖ Minimum cost path condition

- Links with inflows should be on the minimum path

$$\begin{cases} c_{ij}^{s} + \tau_{i}^{s} - \tau_{j}^{s} = 0 & \text{If} \quad y_{ij}^{s} > 0 \\ c_{ij}^{s} + \tau_{i}^{s} - \tau_{j}^{s} \geq 0 & \text{If} \quad y_{ij}^{s} = 0 \end{cases}$$

- c_{ij}^{s}: travel time on link *ij* for users departing at time *s*
- $\tau_i(s)$: Arrival time at node *i* for users departing at time *s*

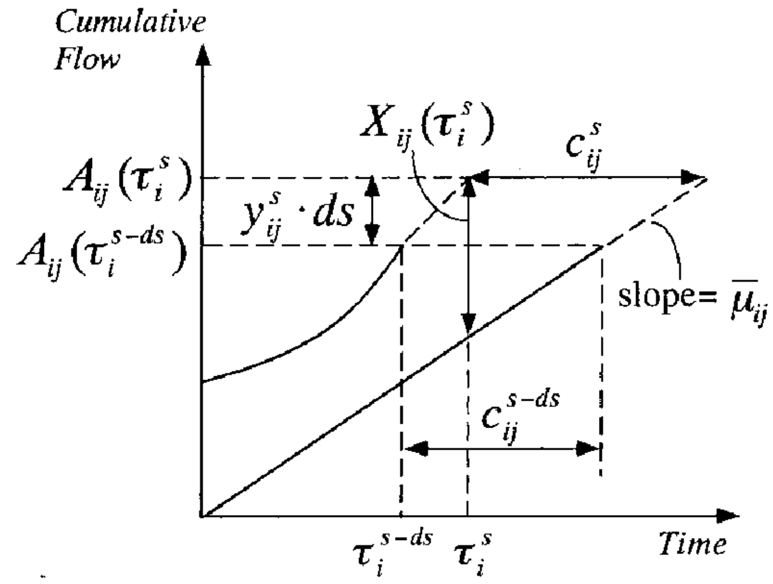
(c) Link travel time

❖ Point queue model with FIFO

- FIFO ⇒ Travel time = Horizontal distance of Cum curves
- The travel time on link *ij* for users departing at time *s*

$$c_{ij}^{s} = \int_{0}^{s} \frac{\mathrm{d}c_{ij}^{t}}{\mathrm{d}t}\mathrm{d}t + c_{ij}^{s=0}$$

$$\frac{\mathrm{d}c_{ij}^{s}}{\mathrm{d}s} = \begin{cases} \frac{y_{ij}^{s}}{\overline{\mu}_{ij}} - \frac{\mathrm{d}\tau_{i}^{s}}{\mathrm{d}s} & \text{If there is a queue} \\ 0 & \text{Otherwise} \end{cases}$$

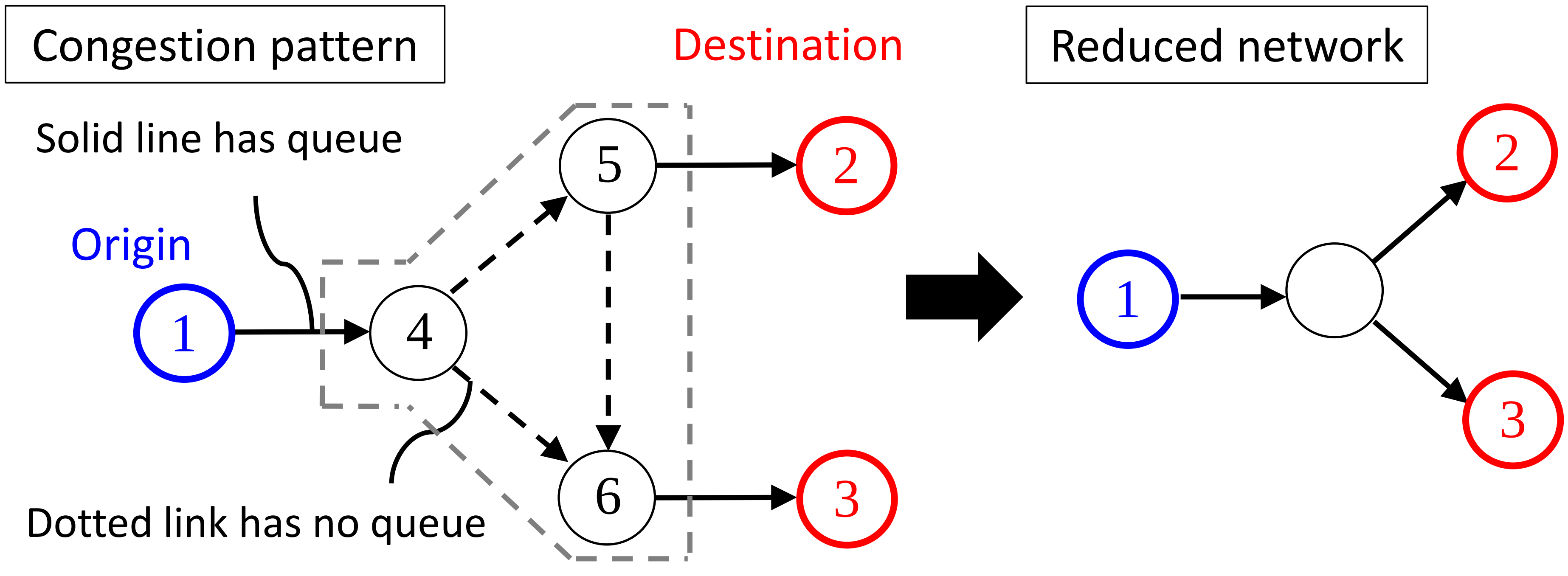


Saturated Network

- ❖ Decomposed formulation of DUE is
  - Formulated as variational inequality problem (VIP)
    - Can not be solved analytically
- ❖ Saturated network [Akamatsu & Heydecker, 03a]
    - All links have positive inflows
    - All links have queues
  - DUE is reduced to a system of linear equations
    - Derive DUE solution analytically
  - ✓ This procedure can be applied to a non-saturated network by constructing a "*Reduced Network*"

Reduced Network [Akamatasu & Heydecker, 03b]

- Represent the topology of saturated links
- Constructed by unifying the initial and terminal nodes of each unsaturated link into a single node



DUE analytical solution

❖ Link travel time (1) & Route choice condition (2)

$$\begin{cases} \dot{c}_{ij}^{s} = y_{ij}^{s}/\mu_{ij} - \dot{\tau}_{i}^{s} \cdots (1) \\ \dot{c}_{ij}^{s} + \dot{\tau}_{i}^{s} - \dot{\tau}_{j}^{s} = 0 \cdots (2) \end{cases} \longrightarrow \begin{array}{l} y_{ij}^{s} = \mu_{ij}\dot{\tau}_{j}^{s} \\ \Leftrightarrow \mathbf{y}^{s} = -(\mathbf{M}\mathbf{A}_{-}^{T})\dot{\boldsymbol{\tau}}^{s} \end{array}$$

❖ Flow conservation

DUE analytical solution

$$\mathbf{A}\mathbf{y}^{s} = -(\dot{\mathbf{Q}}^{s} + \delta) \longrightarrow \boxed{\dot{\boldsymbol{\tau}}^{s} = (\mathbf{A}\mathbf{M}\mathbf{A}_{-}^{T})^{-1}(\dot{\mathbf{Q}}^{s} + \delta)}$$

- $\mathbf{A}$: Node-link incident matrix $\qquad \mathbf{M} \equiv \mathrm{diag}[\cdots, \mu_{ij}, \cdots]$
- δ : column vector whose components are link capacity connecting origin

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Inverse problem of DUE

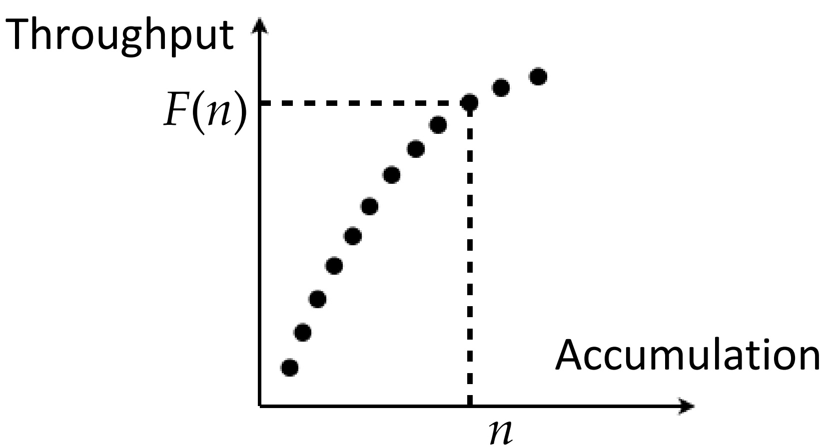
❖ Construct an inverse problem

- Input: Congestion Pattern (A, M)
- Output: Network throughput (OD consistent with AMA)

✓ Alternative condition to OD demand is necessary

❖ Periodic boundary condition (steady state)
[Mazloumian et al.10, Daganzo et al. 11]

- A fixed number of users circulate in a network



$$\dot{\tau}_d^s = 1$$

Network throughput

❖ One-to-many under steady state

$$\mathbf{f} \equiv \mathbf{V}_{dd}\mathbf{1} - (\mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}[\mathbf{V}_{id}\mathbf{1} - \boldsymbol{\delta}_i] + \boldsymbol{\delta}_d)$$

$$\mathbf{V}_{ab} \equiv \mathbf{A}_a\mathbf{M}\mathbf{A}_b$$

i, d: block matrices w.r.t. transient and destination node

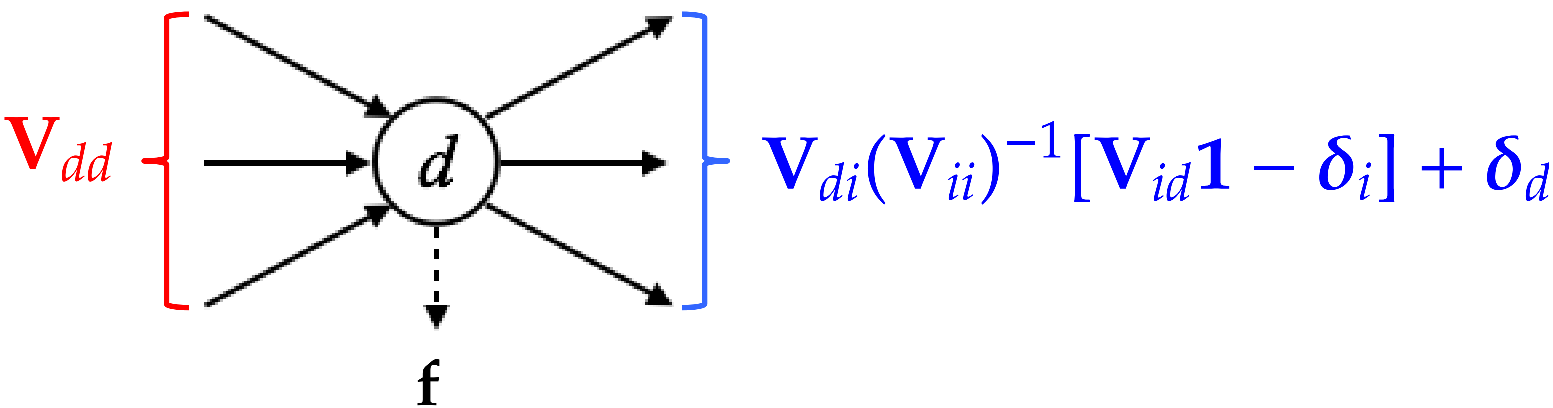
- Characterized by structure of the reduced network
  - Topology, Capacity pattern, OD distribution

Network throughput

❖ One-to-many network under steady state

$$\mathbf{f} \equiv \mathbf{V}_{dd}\mathbf{1} - \left(\mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}\underbrace{[\mathbf{V}_{id}\mathbf{1} - \boldsymbol{\delta}_i]}_{\dot{\tau}^s} + \boldsymbol{\delta}_d\right)$$

- 1^{st} term: Inflows to destinations
- 2^{nd} term: flows passing destinations to transient nodes
  - Including the global effect of the user's route choice



Network throughput

❖ One-to-many network under dynamic state

$$\bar{F} \equiv \mathbf{T}^{-1}\mathbf{V}_{dd}\bar{\dot{\boldsymbol{\tau}}}_d$$

$$- \mathbf{T}^{-1}(\mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}[\mathbf{V}_{id}\bar{\dot{\boldsymbol{\tau}}}_d - \boldsymbol{\delta}_i] + \boldsymbol{\delta}_d)$$

$\mathbf{T} \equiv \mathrm{diag}[\cdots, \bar{\dot{\tau}}_d(\mathbf{x}), \cdots]$

$\bar{\dot{\tau}}_d(\mathbf{x})$: Ave. of $\dot{\tau}_d(\mathbf{x})$ between depart time $s \sim s + \Delta s$

- Dynamic: vehicle accumulation could change
- Network throughput are characterized by
  - Structure of reduced network
  - The travel times between origin and destinations

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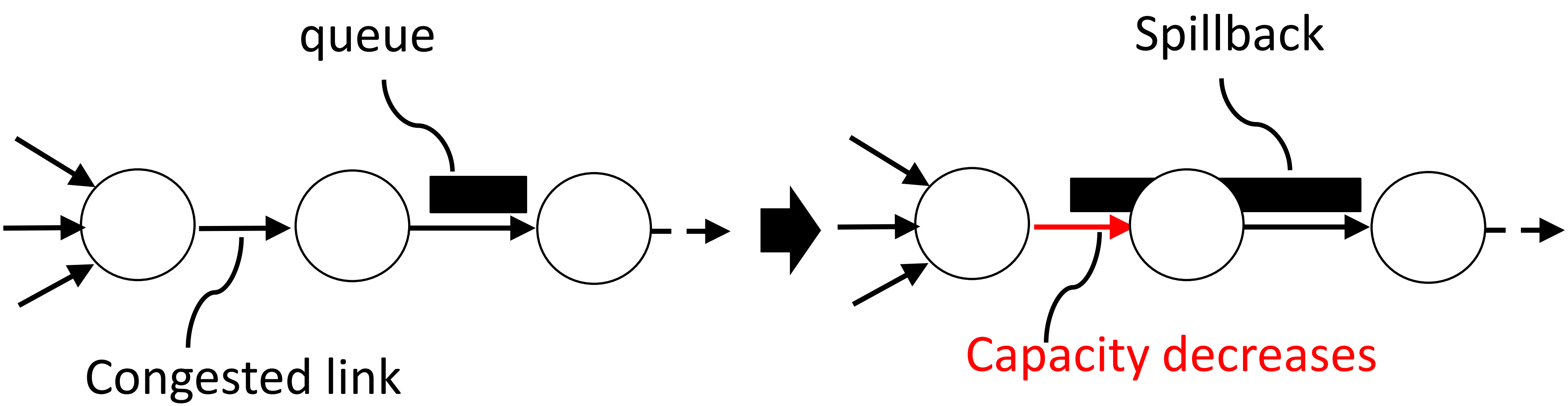
- ❖ Formulation of the decomposed DUE problem
- ❖ Analytical formulas of network throughput
- ❖ Sensitivity analysis and throughput decreasing mechanisms
- ❖ Capacity increasing paradox and network signal control
- ❖ Numerical examples

Sensitivity analysis

- Sensitivity of the network throughput
  - Investigate what type of queue spillback decreases network throughput
    - Spillback: decrease in link capacity



Conditions for decreasing throughput

- Sensitivity coefficient: $\partial F / \partial \mu_{kl}$
  - Case analysis: the types of upstream and downstream node

$$\frac{\partial F}{\partial \mu_{kl}} = \begin{cases} \mathbf{1}^T(\mathbf{A}_d \mathbf{I}_{kl} \mathbf{A}_{d-})\mathbf{1} & \text{if} \quad k,l \in \mathcal{N}_d \quad \text{or} \quad k = o \ \wedge \ l \in \mathcal{N}_d \\ -\mathbf{1}^T \mathbf{e}_k & \text{if} \quad k \in \mathcal{N}_d \ \wedge \ l = o \\ -\mathbf{1}^T \mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}(\mathbf{A}_i \mathbf{I}_{kl} \mathbf{A}_{i-})\dot{\boldsymbol{\tau}}_i & \text{if} \quad k = o \ \wedge \ l \in \mathcal{N}_i \quad \text{or} \quad k,l \in \mathcal{N}_i \\ \mathbf{1}^T \mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}\mathbf{e}_k & \text{if} \quad k \in \mathcal{N}_i \ \wedge \ l = o \\ \mathbf{1}^T(\mathbf{A}_d \mathbf{I}_{kl} \mathbf{A}_{d-})\mathbf{1} - \mathbf{1}^T \mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}(\mathbf{A}_i \mathbf{I}_{kl} \mathbf{A}_{d-})\mathbf{1} & \text{if} \quad k \in \mathcal{N}_i \ \wedge \ l \in \mathcal{N}_d \\ \mathbf{1}^T(\mathbf{A}_d \mathbf{I}_{kl} \mathbf{A}_{i-})\dot{\boldsymbol{\tau}}_i - \mathbf{1}^T \mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}(\mathbf{A}_i \mathbf{I}_{kl} \mathbf{A}_{i-})\dot{\boldsymbol{\tau}}_i & \text{if} \quad k \in \mathcal{N}_d \ \wedge \ l \in \mathcal{N}_i \end{cases}$$

  - $\partial F / \partial \mu_{kl} > 0$: Network throughput decreases when the link capacity decreases
    - Identify conditions which decrease throughput

Conditions for decreasing throughput

❖ Conditions for network throughput decreasing

- Due to the changes of 1st term or 2nd term

$$\mathbf{f} \equiv \mathbf{V}_{dd}\mathbf{1} - (\mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}[\mathbf{V}_{id}\mathbf{1} - \boldsymbol{\delta}_i] + \boldsymbol{\delta}_d)$$

i. (origin, destination) :

- Decrease 1st term

ii. (origin, transient) or (transient, transient) : Conditionally

- Increase 2nd term

iii. (transient, destination)

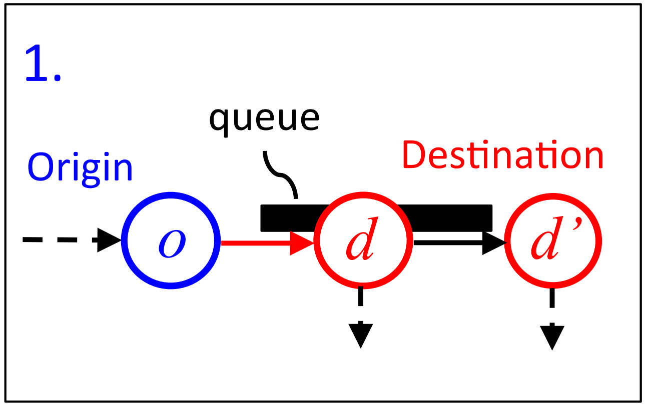
- Decrease 1st term and 2nd term (former is stronger)

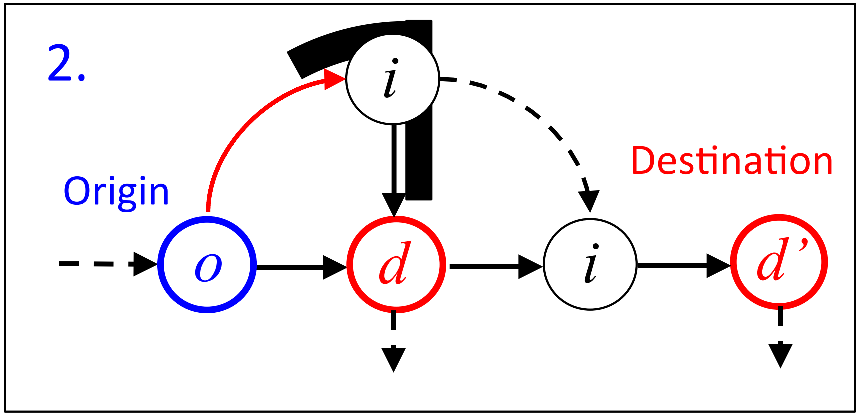
Decreasing mechanisms of throughput

❖ Two types of queue spillback

1. Blocking: decreasing 1st term
   - Prevents flows from entering destinations
2. Alters route choice pattern: changing 2nd term
   - Increasing of flows passing destinations

➢ Decreasing network performance is caused by the interaction between users with different destinations





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Capacity increasing paradox

- Capacity increasing paradox
  - Increase in the capacity of a certain link leads to a decrease in the network throughput
    - Identify conditions: $\partial F / \partial \mu_{kl} < 0$

- Conditions for the paradox:
  i. (destination, origin)
  ii. (transient, origin) or (transient, transient) : Conditionally
  iii. (destination, transient)
  - Regulate these links $\rightarrow$ network performance increase

  ✓ Difficult to control a single link independently

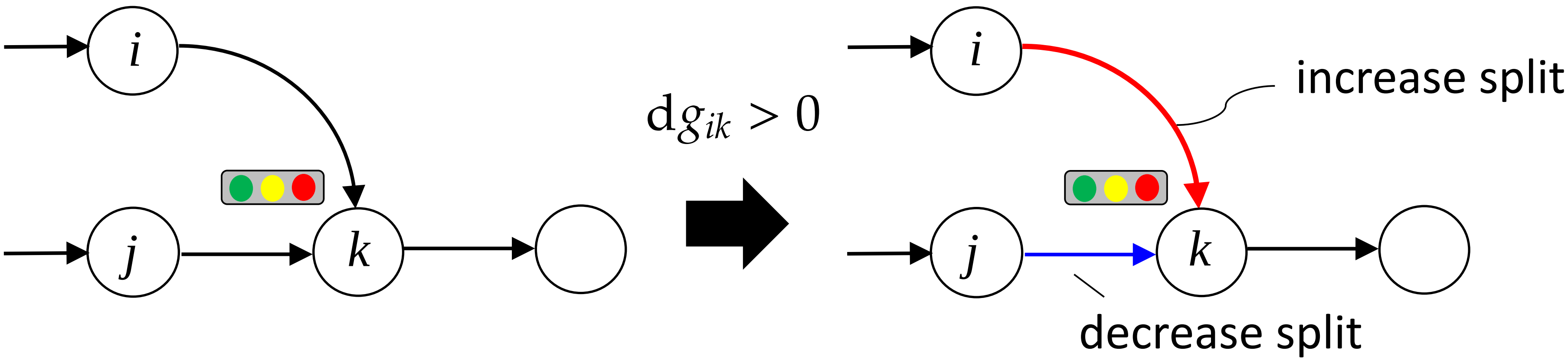
Signal control

❖ Sensitivity when green splits change at a saturated merge

- Capacity = saturation flow rate $s_{ij}\times$ split g_{ij}
- Identify the condition: $\mathrm{d}g_{ik} > 0 \to \mathrm{d}F > 0$

$$\mathrm{d}F = \frac{\partial F}{\partial \mu_{ik}}\mathrm{d}\mu_{ik} + \frac{\partial F}{\partial \mu_{jk}}\mathrm{d}\mu_{jk} = \frac{\partial F}{\partial \mu_{ik}}(s_{ik}\mathrm{d}g_{ik}) + \frac{\partial F}{\partial \mu_{jk}}(s_{jk}\mathrm{d}g_{jk})$$

$$= \left(s_{ik}\frac{\partial F}{\partial \mu_{ik}} - s_{jk}\frac{\partial F}{\partial \mu_{jk}}\right)\mathrm{d}g_{ik} \qquad \left(\mathrm{d}g_{ik} = -\mathrm{d}g_{jk}\right)$$



Signal control

❖ Signal control at congested transient node

No.	upstream nodes	dF	causes of sensitivity	additional condition
1	$i = o \wedge j \in \mathcal{N}_i$	≥ 0	second term	–
2	$i = o \wedge j \in \mathcal{N}_d$	> 0	second term	–
3	$i, j \in \mathcal{N}_i$	≥ 0	second term	there is no route from the origin to node i so that it passes thorough a destination.
4	$i \in \mathcal{N}_i \wedge j \in \mathcal{N}_d$	≥ 0	second term	–
5	$i, j \in \mathcal{N}_d$	0	second term	–

- Strategy: decrease the through traffic of destinations
  - Upstream = origin: increase the split of the link
  - Upstream = destination: decrease the split

Signal control

❖ Signal control at congested destination node

No.	upstream nodes	dF	causes of sensitivity	additional condition
1	$i = o \ \wedge j \in \mathcal{N}_i$	> 0	first & second terms	$s_{ik} - s_{jk} > 0.$
2	$i = o \ \wedge j \in \mathcal{N}_d$	> 0	first term	–
3	$i, j \in \mathcal{N}_i$	> 0	first & second terms	$s_{ik} - s_{jk} > 0$
4	$i \in \mathcal{N}_i \ \wedge \ j \in \mathcal{N}_d$	≥ 0	first & second terms	–
5	$i, j \in \mathcal{N}_d$	0	first term	–

- Strategy: increase inflows to destinations
  - Upstream ≠ dest: increase the split
  - Both upstream ≠ dest: increase the link having HIGH s_{ij}

➢ Feature: determines adjustment directions of splits from only local information → local and distributed signal control

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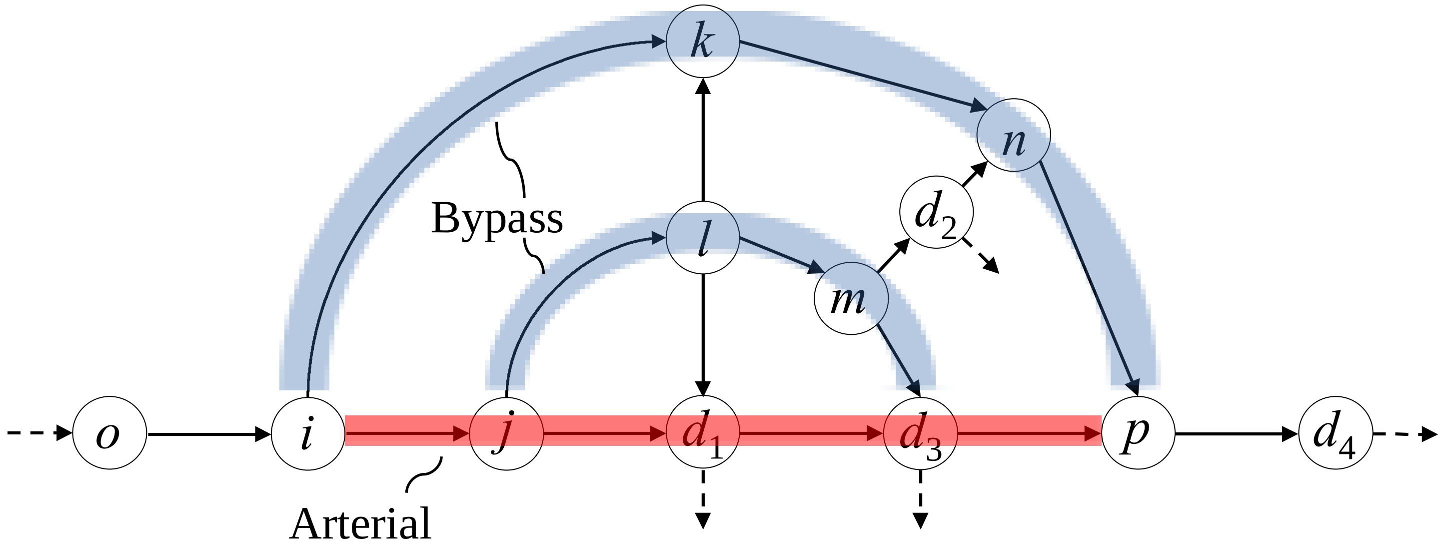
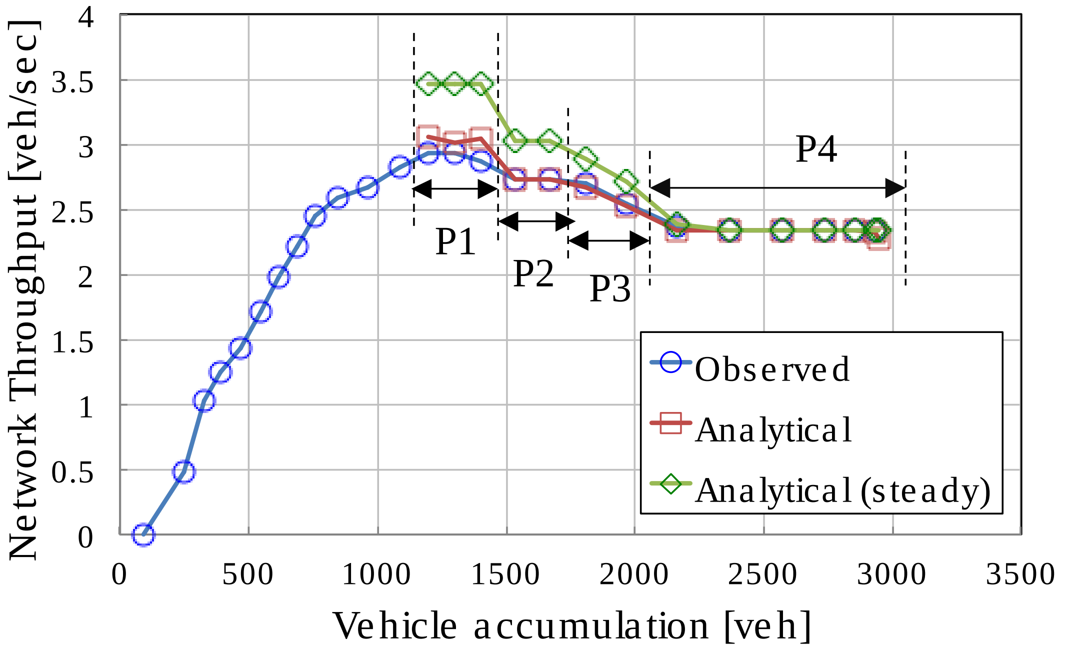
Numerical example: Settings



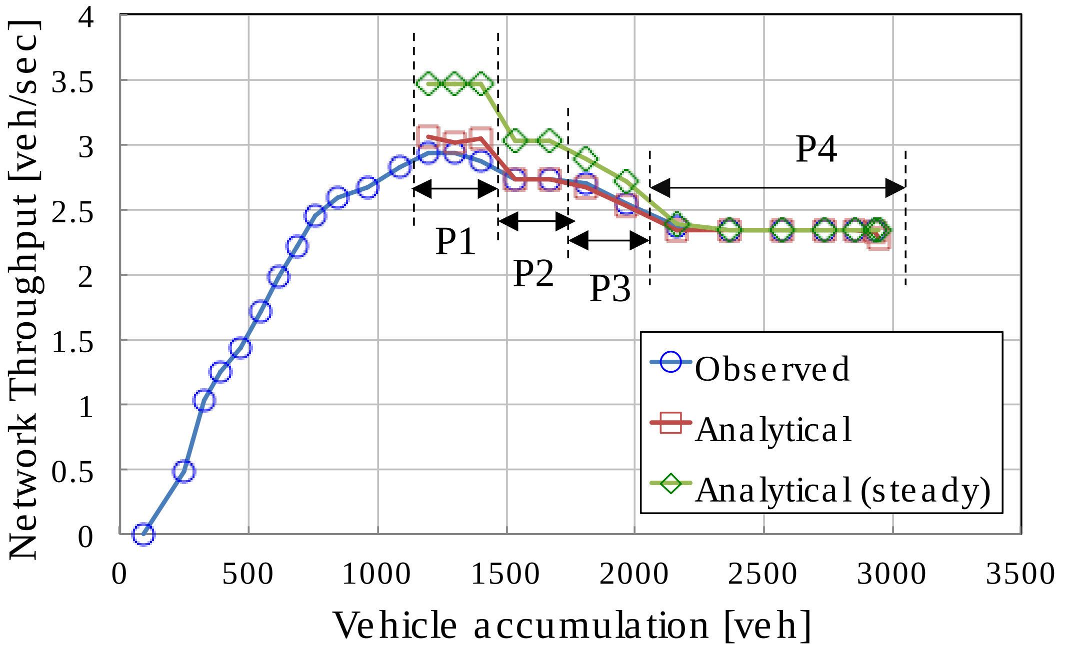
- ❖ Validate the proposed method
  - Calculate the accurate DUE according to Iryo (2011)
  - Aggregate traffic variables every 3minutes

Results: network throughput

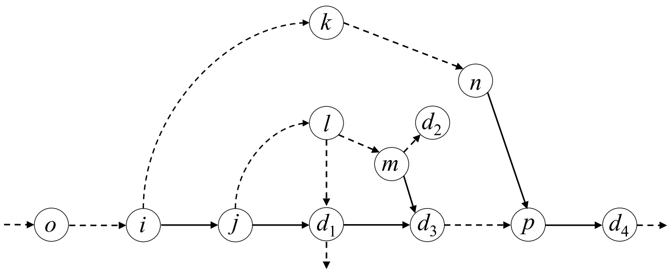


- Dynamic: agree well with observed ones
- Steady: tend to overestimate
  - Capture the decreasing behavior of throughputs

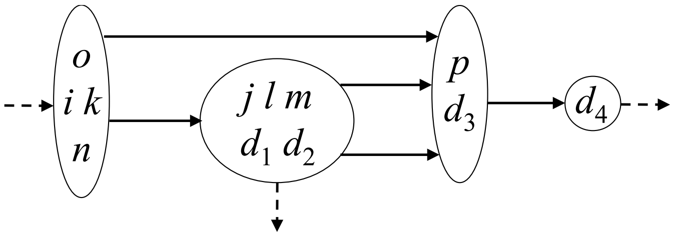
Results: network throughput



Original

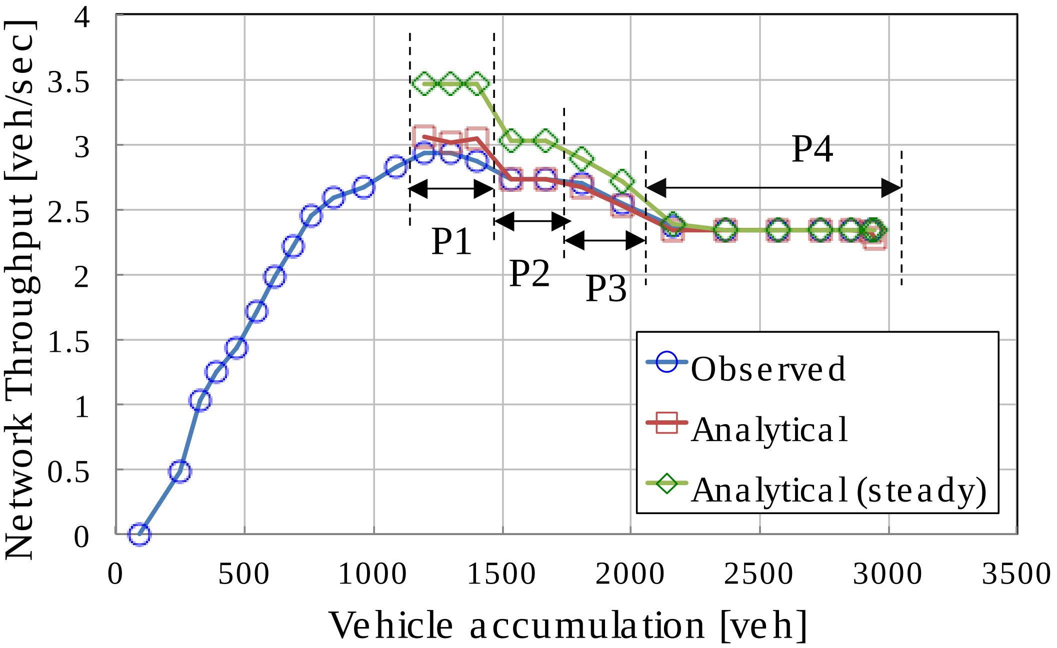


Reduced



- ❖ Pattern 1 (Solid: saturated Dotted: not saturated)
  - All destinations are separated on the reduced network

Results: network throughput



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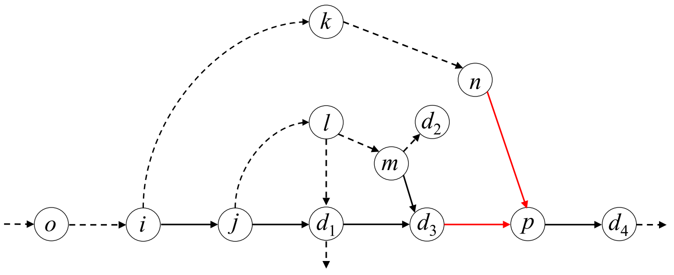
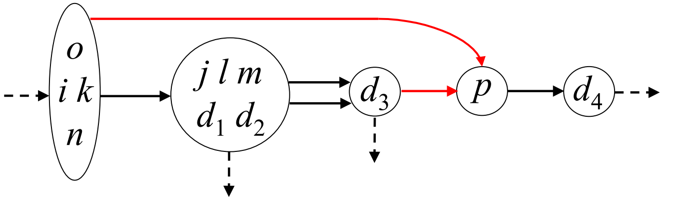
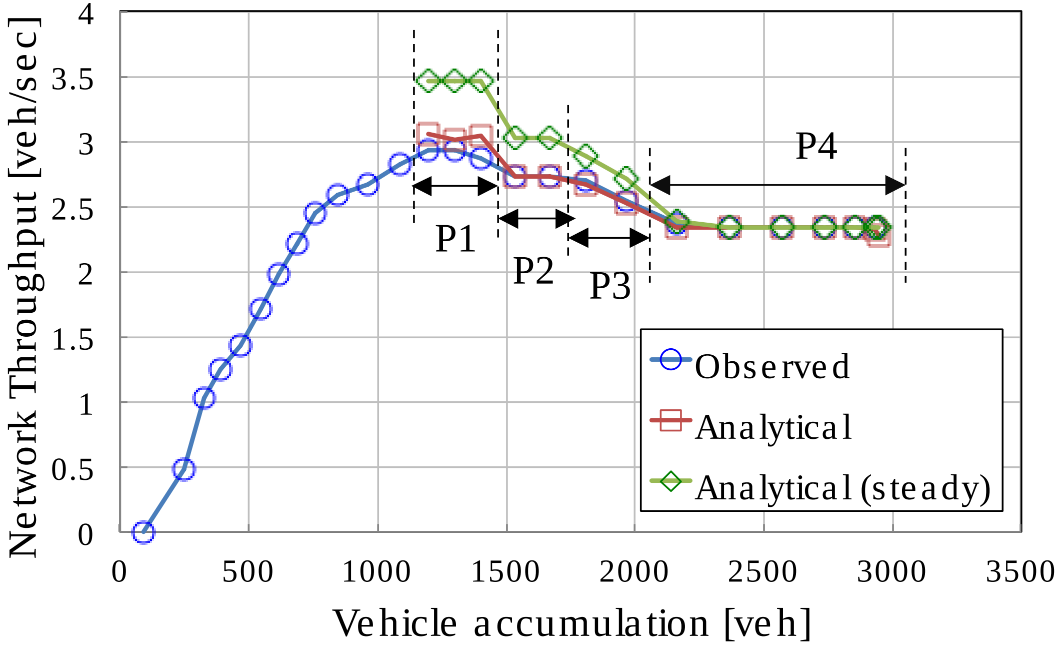


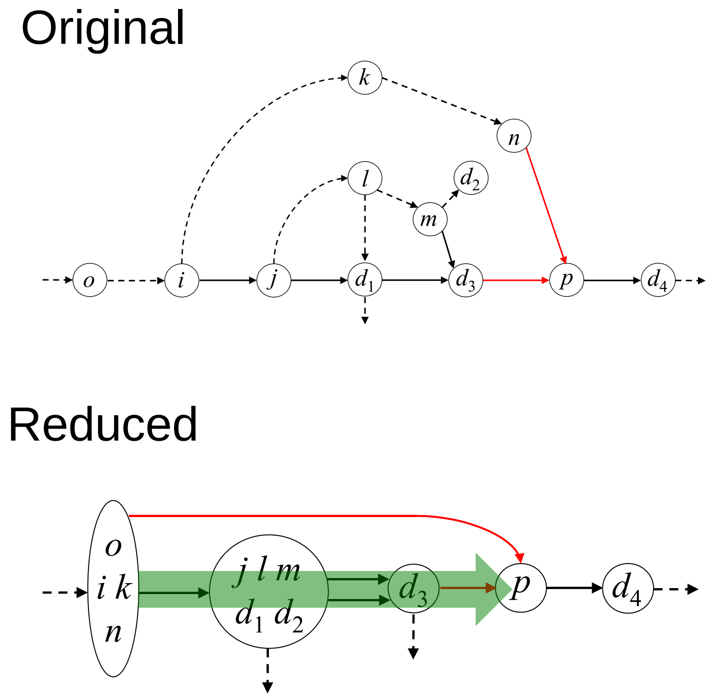
Reduced



- ❖ Pattern 2 (Red: queue spillback)
  - Several link capacities decrease due to queue spillback
  - Increase the through traffic → throughput decreases

Results: network throughput





❖ Pattern 2 (Red: queue spillback)

- Several link capacities decrease due to queue spillback
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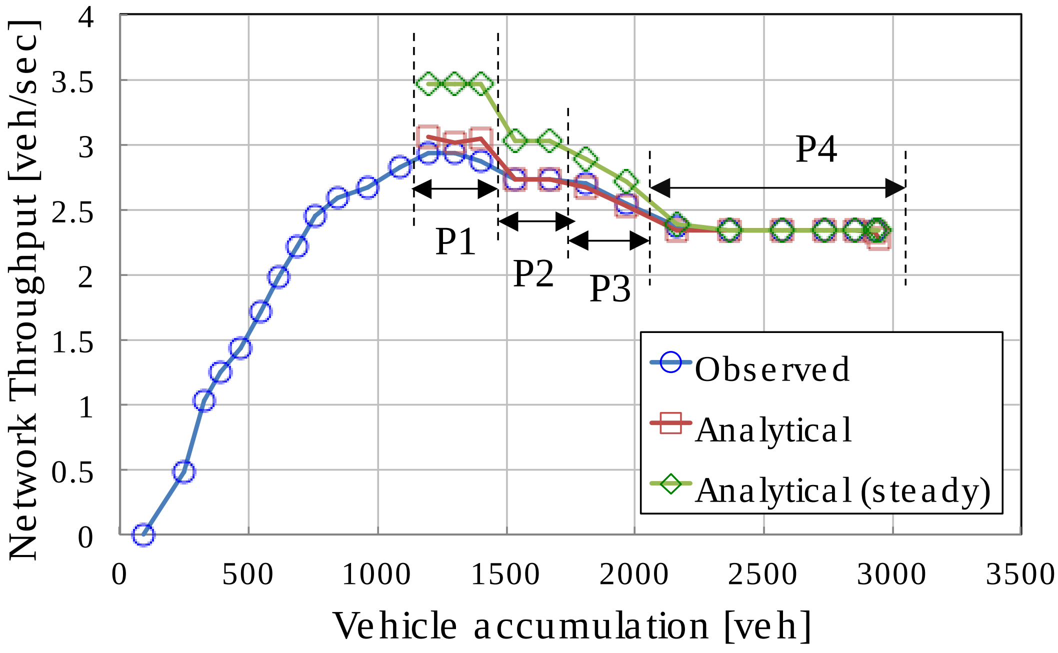
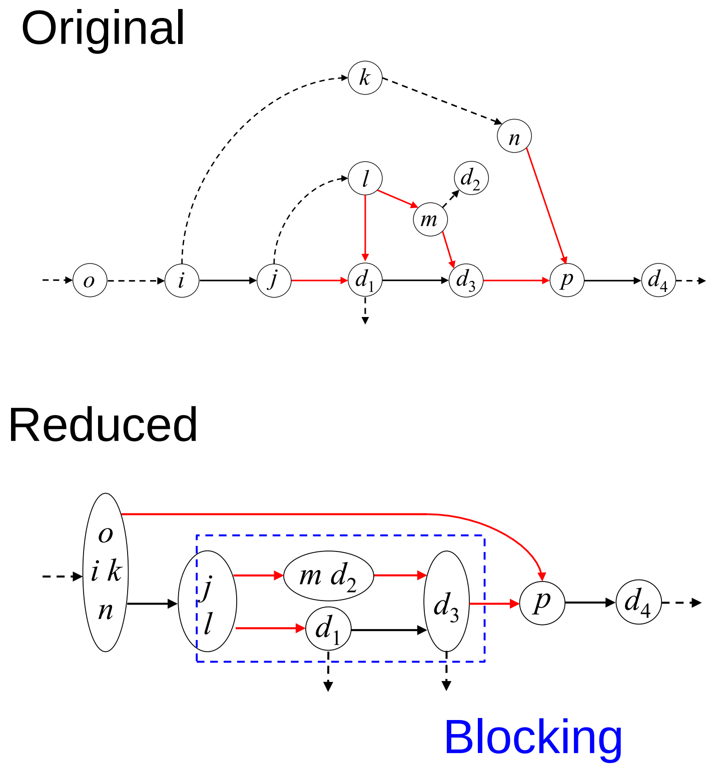
Results: network throughput





❖ Pattern 3

- Blocking: queue spillback decreases flows exiting to destination

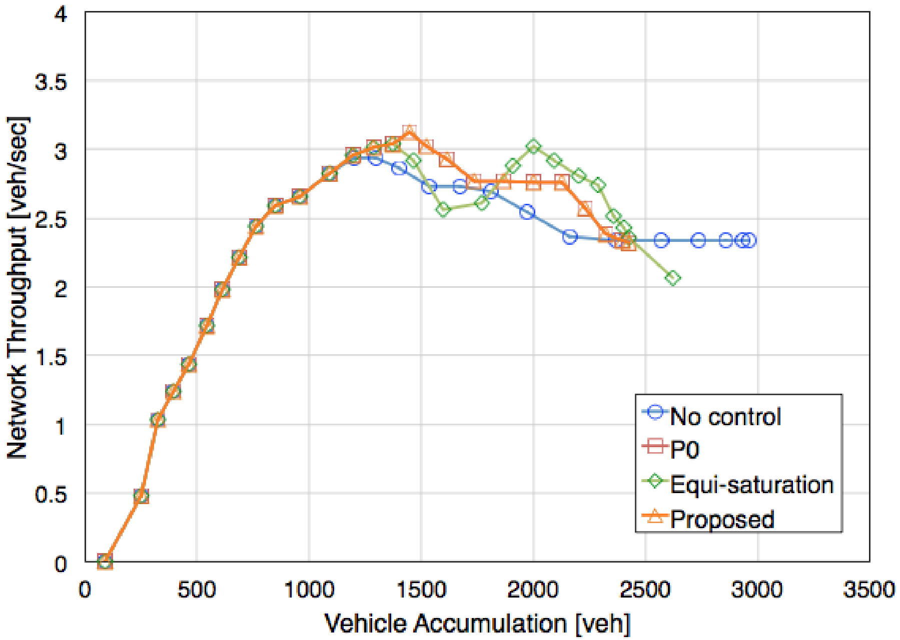
➢ Analyzed decreasing mechanisms are valid

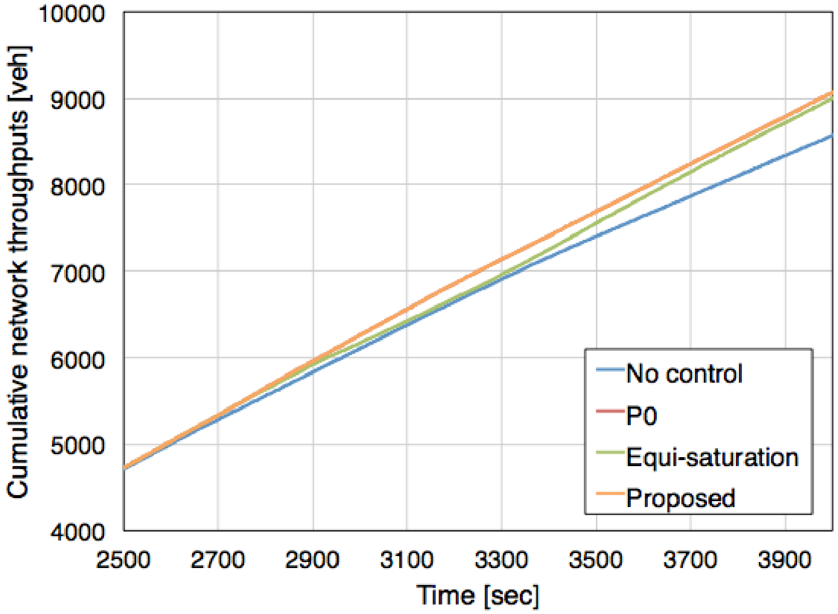
Numerical examples: signal control

- Investigate the effectiveness of the signal controls
  - Compare three signal controls

1. Equi-saturation Policy [Webster (1958)]
   - More delayed link has more green split
2. Policy P0 [Smith (1979)]
   - More pressured link has more green split
   - Pressure = (saturation flow rate) × (queuing delay)
3. Proposed policy

Results: comparison strategies





- Policy P0 and the proposed policy
  - Achieve higher network throughputs than the others
- Equi-saturation: oscillate with the increase in vehicle accumulation

Summary

- Derive the analytical formula of network throughput for a given congestion pattern
  - Derived by solving the inverse problem of the DUE
  - Incorporate the effect of structure of reduced network
    - Topology, Capacity pattern, OD distribution

      Travel time (dynamic)
- Conduct the sensitivity analysis
  - Clarify decreasing mechanisms of network throughput
    - Blockage, Increasing of flows passing through destination
  - Identify the conditions for occurrence of the paradox
  - Propose signal control strategy based on congestion pattern

Future plan

❖ Validate the signal control strategy

- Systematic numerical experiments are necessary
- Clarify the relationship btw proposed and Policy P0

❖ Extend the method to many-to-many network

- Expectation: Not need to treat a complex VIP for DUE
  - congestion pattern is given
- Idea: Cyclic decomposition approach
  - Decompose many-to-many into one-to-many