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Optimization of Transit Timetable Considering Transit Assignment

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Abstract

In the public transportation system, transit synchronization may require in order to travel to their desired destinations. Designing an efficient transit timetable to meet varied passenger demand with minimal travel time is very challenging. In this paper, a mixed-integer programming (MIP) model that aims to minimize passenger travel time is proposed using dynamic assignments. To verify the efficiency of the model, a case study that contains eight scenarios were demonstrated with considering of frequency, direction of passenger flows and walking time in the station. The results of the case study show that our MIP model provides the reasonable results and has potential to improve the performance of transit systems. Optimality gaps obtained by MIP increase among the problem difficulties.

Keywords: Transit assignment; Timetabling; Mixed-integer programming

Nomenclature

A_l	set of physical network links *i* to *j* which bus on line *l* passing
A_{gi}	set of physical and dummy network links *i* to *j* which bus on line *l* passing for passenger group *g*
G	set of original, destination and starting time of passenger group
L	set of physical line indexed by l excluding dummy link

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$t^{-1}(g)$	set of departure time of passengers
S	set of physical stations
X_{gij}	binary variable. 1, if passenger group travel from i to j; 0, otherwise
Z_{gslr}	binary variable. 1, if the passenger group g transfers from line l to line r at station s; 0, otherwise
A_{gs}^{P}	continuous variable denotes arrival time of passenger group *g* at the station *s*
D_{iln}^{B}	continuous variable denotes departure time of bus n^{th} on line *l* at station *i*
R_{ijln}	continuous variable denotes running time of bus nth on line *l* moving from *i* to *j*
H_{iln}	continuous variable denotes headway of bus n^{th} on line *l* at station *i*
S_{gs}	continuous variable denotes time of passenger group g staying at station *s*
D_{gs}^{P}	continuous variable denotes departure time of passenger group *g* at the station *s*
W_{gs}	continuous variable denotes waiting time of passenger group *g* at station *s*
d	dwelling time
f_l	number of buses in line *l*
h^{ub}, h^{lb}	upper and lower bound of headway
k	maximum number of total transfers
M	big number
n_g	number of passenger group *g*
p	penalty of unserved passengers
q	maximum initial waiting time
r^{ub}, r^{lb}	upper and lower bound of running time of bus moving from *i* to *j*
τ_{gs}	transfer time of passenger group *g* at station *s*
u_g	upper bound of travel time

1. Introduction

In the public transportation systems, passengers may have to make transfers to travel to their desired destinations. Transfers are undesired because passengers may have to wait for a considerable amount of time for transfers and deal with risk of missing the connection. Designing an efficient transit timetable to meet passenger demand is very challenging. In the literature studies proposed various techniques to create and develop timetables with different objectives for specific situations such as Wong, Yuan, Fung and Leung (2008), Rojas and Rois-Solis (2012) and Niu, Tian and Zhou (2015). However, most of these studies assumed that the transfer demands are constant, which is unrealistic because passengers clearly make travel decisions based on the number and the time required for transfers in their trips. Therefore, the main motivation of the study is the consideration of the responses of passengers to timetables.

In this paper, an MIP optimization model is proposed for scheduling transit timetable which minimizes passenger time to improve service level. The major contribution of this study is the use of variable in demand variation including the vehicle capacities restriction. Passenger route choices and timetables are determined separately.

2. Mathematical model

Given a graph representing the transit network excluding bus depot, a bus route is defined as a sequence of network nodes which a bus travels with specified directions. Bus fleet sizes and capacities are fixed for different bus lines while bus frequencies are constant. Bus timetable is the outcome of the bus routes and frequencies. A group of passenger with the same origin and destination (OD) will be defined as an OD pair. Passenger demands from origin and destination including public transportation network are the input data and will be assigned to the feasible passenger path with satisfies the bus capacity constraints. The major simplifying assumptions adopted from James C. Chu, 2018 are stated as follows:

2.1. Mixed integer model formulation

A transit network consists of many physical lines. A dummy line without an actual bus is added for unserved passengers. Moreover, a dummy station is allocated for distinguishing passenger flow on the dummy lines and the physical lines. All unserved passengers travel from the origin through the dummy station and the sink station while others travel via physical lines. The vehicle running time on the dummy line will be zero except the link from the origin to the dummy station. The objective function is to minimize total travel time of passengers including the penalty for unsatisfied demands.

- Timetable connection

$$D^B_{jln} = D^B_{iln} + R_{ijln} + d, \forall l \in L, (i,j) \in A_l, n \in \{1,.., f_l\} \tag{2}$$

$$D^B_{iln} = D^B_{il(n-1)} + H_{iln}, \forall l \in F, i \in S_l, n \in \{2,.., f_l\} \tag{3}$$

$$h^{lb} \le H_{iln} \le h^{ub}, \forall l \in L, i \in S_l, n \in \{1.. f_l\} \tag{4}$$

$$r^{lb}_{ij} \le R_{ijln} \le r^{ub}_{ij}, \forall l \in L, (i,j) \in A_l \tag{5}$$

$$R_{ij01} = p, \forall g \in G, (i,j) \in A_{g0} \tag{6}$$

Constraints (2) and (3) ensure timetable connection between lines. For each line l, constraint (2) states the consistency of the route in each line which implies that the departure time of a bus in station j is the summation of the departure time of the previous station, bus running time from the previous station to station j and dwelling time at station j. Constraint (3) represents the connection of each departure time. The limit of headway and the variation of bus running time between stations in each line are stated in constraint (3) and (4). Constraint (5) ensures all the travel time in dummy line are zero. Finally, constraint (6) represents the penalty of unserved passengers.

- Feasible passenger path

Based on the study assumptions, passengers will choose their routes based on shortest travel time. Network flow constraints are adopted for passenger path generation. Constraints (7) – (9) ensure the flow conservation of each passenger group. Constraint (10) forces passenger groups that depart from dummy station at time $t^{-1}(g)$ while constraint (11) is departure time of passenger group. In case of long journey, the maximum travel time that passengers are willing to take transit service instead of private car will be assumes as double shortest possible travel time between same OD pair. Another is for short trip; travel time of all passengers should be less than shortest possible travel time between same OD pair and maximum initial wait time. Feasible passenger path conditions are stated by constraints (12) – (16).

$$\sum_{(i,j)\in E} X_{gij} = \sum_{(j,k)\in E} X_{gjk}, \forall j \in S, g \in G, j \not\subset o^{-1}(g) \cup b \tag{7}$$

$$\sum_{(o^{-1}(g),s)\in E} X_{g,o^{-1}(g),s} = 1, \forall g \in G \tag{8}$$

$$\sum_{(s,b)\in\{d^{-1}(g),b\}\cup\{a,b\}} X_{gsb} = 1, \forall g \in G \tag{9}$$

$$A^P_{o^{-1}(g)} = t^{-1}(g), \forall g \in G \tag{10}$$

$$A^P_{gs} + S_{gs} = D^P_{gs}, \forall s \in S, g \in G \tag{11}$$

$$A^p_{gb} - t^{-1}(g) \le u_g, \forall g \in G \tag{12}$$

$$u_g = max\left\{2 \times b_{o^{-1}(g),d^{-1}(g)}, b_{o^{-1}(g),d^{-1}(g)} + q\right\}, \forall g \in G \tag{13}$$

$$\sum_{i\in S,(l,r)\in P} Z_{gilr} \le k, \forall g \in G \tag{14}$$

$$W_{gs} \le w, \forall g \in G, s \in S, s \ne o^{-1}(g), s \ne d^{-1}(g) \tag{15}$$

$$S_{g,o^{-1}(g)} \le q, \forall g \in G \tag{16}$$

$$-M \times \left(1 - \sum_{(l,r)\in P_s} Z_{gslr}\right) + S_{gs} \le \tau_{gs} + W_{gs}, \forall s \in S, g \in G \tag{17}$$

$$-M \times \left(1 - \sum_{(l,r)\in P_s} Z_{gslr}\right) + S_{gs} \le \tau_{gs} + W_{gs}, \forall s \in S, g \in G \tag{18}$$

- Integration of passengers and bus schedules

Constraint (19) forces passenger group boarding on one of bus lines while constraints (20) – (21) match the passenger arrival time at station with next approaching bus. Constraints (22) – (23) state passenger arrival time at next station. Moreover, passenger arrival time is calculated by summation of passenger departure time and bus running time between station. Finally, constraints (24) – (25) allow the transferring variable Z_{gilr} if a passenger decides to take another bus to leave stop *i*.

$$X_{gij} = \sum_{l\in L\cup\{d\},(i,j)\in A_l, n\in\{1,..,f_l\}} Y_{gijln}, \forall g \in G, (i,j) \in E_g \tag{19}$$

$$-M \times \left(1 - Y_{gijln}\right) + D_{gi}^{P} \le D_{iln}^{B}, \forall i \in S, l \in L, n \in \{1,..,f_l\}, g \in G, (i,j) \in A_l \tag{20}$$

$$M \times \left(1 - Y_{gijln}\right) + D_{gi}^{P} \ge D_{iln}^{B}, \forall i \in S, l \in L, n \in \{1,..,f_l\}, g \in G, (i,j) \in A_l \tag{21}$$

$$M \times \left(1 - Y_{gijln}\right) + A_{gj}^{P} \ge R_{ijln} + D_{gi}^{P}, \forall g \in G, l \in L \cup \{0\}, (i,j) \in A_{gl}, n \in \{1,..,f_l\} \tag{22}$$

$$-M \times \left(1 - Y_{gijln}\right) + A_{gj}^{P} \le R_{ijln} + D_{gi}^{P}, \forall g \in G, l \in L \cup \{0\}, (i,j) \in A_{gl}, n \in \{1,..,f_l\} \tag{23}$$

$$\sum_{l\in L,(j,i)\in A_l, n\in\{1,..,f_l\}} Y_{gjiln} \ge Z_{gilr}, \forall g \in G, i \in S, (l,r) \in P_i \tag{24}$$

$$\sum_{r\in L,(i,q)\in A_l, n\in\{1,..,f_r\}} Y_{giqrn} \ge Z_{gilr}, \forall g \in G, i \in S, (l,r) \in P_i \tag{25}$$

$$\sum_{r\in L,(i,q)\in A_l, n\in\{1,..,f_r\}} Y_{gifrn} + \sum_{l\in L,(j,i)\in A_l, n\in\{1,..,f_l\}} Y_{gjilv} \le Z_{gilr} + 1, \forall g \in G, i \in S, (l,r) \in P_i \tag{26}$$

$$V_{ijln} = \sum_{g\in G} \left(Y_{gijln} \times n_g\right), \forall l \in L, (i,j) \in A_l, n \in \{1..f_l\} \tag{27}$$

$$V_{ijln} \le c, \forall l \in L, (i,j) \in A_l, n \in \{1..f_l\} \tag{28}$$

3. Numerical examples

A demonstration case is used to test the efficiency of MIP solutions and evaluate the factors which affect the computational time. The hardware environment is Intel® Core™ i7 3.4GHz CPU and 12 GB RAM on Microsoft Windows 10. Python 2.7 and Gurobi 7.5 were used to obtain optimal solutions. The numerical example is divided into three main scenarios which are low frequency and high frequency. Our demonstrated transit network contains 7 stations and 2 transfer stations which are Station 4 and Station 5 (Fig. 1). Different parameters such as frequency, passenger flow direction and transfer-walking time are adjusted to different degree to investigate the impact and explore the trade-offs among these parameters and how they affect to the computational time. In this example, passenger groups are assumed to transfer only once. Note that the computation time limit is 36 hours. Bus fleet size of line S, S* and line G, G* are 5 while line K, K* are 10. Finally, the maximum waiting time are adjusted to be 100 minutes to avoid the difficulty for solving problems.

4. Result analysis

According to the computational results (Table 1), the low frequency cases were solved within 27.73 hours while in the cases with the high complexity, the solution times require more than 36 hours to solve. For the lower bound of the solution found by Gurobi, the optimality gaps of solutions obtained by MIP are on average 11.69%. Almost all of passengers are served except the low frequency scenario with bi-directional direction and considering transfer-walking time in the station due to the difficulty in order to optimize the low frequency bus timetables with all parameters are taken into consideration. In addition, the computational time of MIP solutions increase sharply for bi-direction and when considering transfer-walking time. For the high frequency scenarios, the solution gaps increase among the problem complexity with the average of 14.52%.

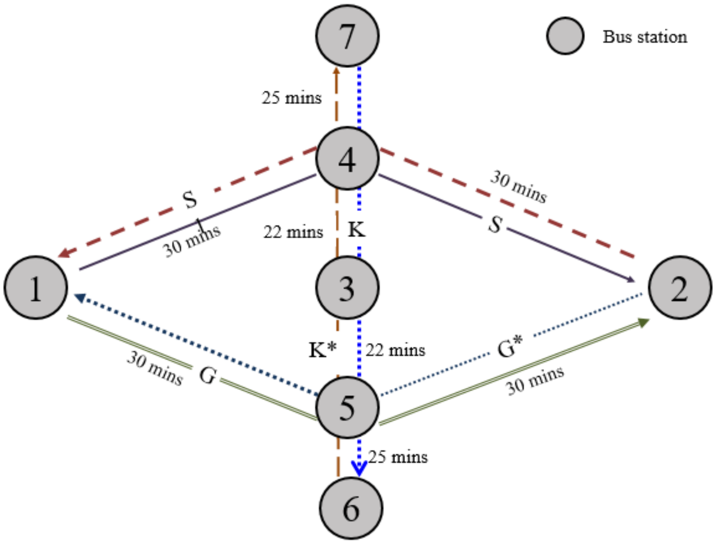

Fig. 1. Demonstrated transit network for numerical example

Table 1. Computational results for different scenarios using MIP.

	Low frequency				High frequency			
Direction	Single		Bi-direction		Single		Bi-direction	
OD no.	6x5		12x5		6x10		12x10	
Walking time	0 mins	9 mins	0 mins	9 mins	0 mins	9 mins	0 mins	9 mins
Objective	12276.0	18064.0	31600.0	45762.0	21085.0	28108.5	44571.5	53686.5
	12276.0	18064.0	31600.0	45762.0	21085.0	24283.8	35974.4	40174.9
Gap	0%	0%	0%	0%	0%	13.6%	19.3%	25.2%
Unserved	1	5	2	17	0	8	0	2
Solution time (s.)	17	321	877	98854	6660	129600	129600	129600

4.1. Optimized passenger paths and synchronization activities

Timetable synchronizations of bi-direction with consideration of transfer walking time are illustrated in Fig. 2. Passenger groups were randomly distributed during the day with different passenger in each group. They travel from Station 3 to 1 and Station 1 to 3. Low frequency and high frequency are 5 and 10 minutes for all bus lines respectively. In Fig 2(a), there are unserved passengers which are 1 groups from OD:3→1 and 2 groups for OD:1→3. These are due to the low bus frequency and passengers have to spend longer time during transfer. Waiting time constraint is extended in order to simplify the problem especially for the low frequency. In contrast, comparing to high frequency as shown in Fig. 2(b), all passengers are served and spend less time for transfer waiting time. It is clear that high frequency case is much more complicated and cannot be solved to obtain the optimal solution in 36 hours. However, only Station 4 is chosen as the interchange station in these cases since the total travel time of the passengers are minimized.

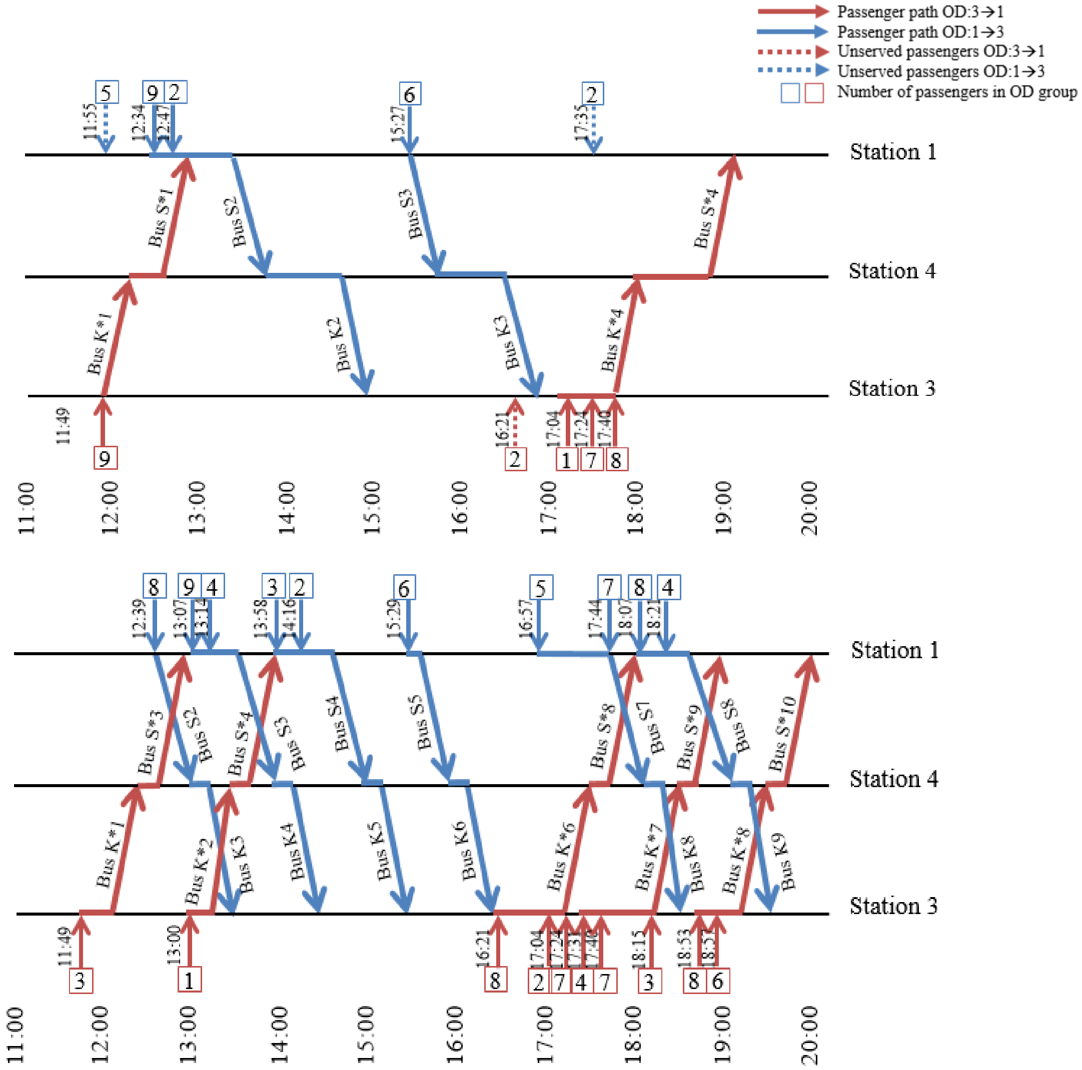

Fig. 2 Passenger paths and transfer activities for OD:3→1 and OD1→3. (a) Low frequency buses with consideration of transfer walking time (b) High frequency buses with consideration of transfer walking time

5. Conclusion

This paper presents the timetable scheduling for transit assignment which aims to utilize passenger satisfaction. The optimized model is formulated as MIP with minimal total travel time of passengers. Main concept of this model formulation is to integrate the bus timetables with passenger flows. Passenger groups are divided into small OD groups due to the limitation of MIP and randomly distributed during the day. Moreover, over highlighted major contribution is the variation of passenger demands. In our numerical examples, the computational time is influenced by passenger flow directions, bus frequencies and passenger transfer walking time. Almost all scenarios reach the optimality excepted the high frequency scenarios with more complexities. Unserved passengers from the low frequency bus services increase especially for bi-direction with consideration of transfer waling time. Finally, according to the computational results, our MIP model formulation is able to solve transit assignment problems efficiently.

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