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Impact of VSL location on capacity drop: A case of sag and tunnel bottlenecks

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Abstract

When there is upstream congestion the discharging flow-rate of a tunnel or sag bottleneck can drop, which leads to additional traffic jams. Therefore, control strategies such as variable speed limit (VSL) have been developed aiming to prevent or mitigate upstream traffic congestion. Understanding traffic dynamics at bottlenecks, especially the mechanism of capacity drop, is critical for developing such models. Many studies are centered on the control algorithm design of VSL. However, there are few studies that systematically anayze the effect that the VSL application area has on the control effectiveness. This paper extends to sag and tunnel bottlenecks the theoretical framework to analytically solve the optimal location of the speed limit application area (first developed in Martínez and Jin (2018)). Moreover, we prove that the optimization formulation can be simplified. Consequently, it can be applied to further bounded acceleration models than the constant one. Finally, for an open-loop control with a constant speed limit for the Kobotonoke tunnel bottleneck, we validate the analytic definition of optimal location by preventing capacity drop in numerical simulations.

Keywords: Sags and tunnels, Capacity drop, Continuum car-following model, variable speed limit, optimal control location

1. Introduction

A complete understanding of the mechanism behind the queue formation that triggers the so-called capacity drop is still elusive (Banks (1991), Hall and Agyemang-Duah (1991), Cassidy and Bertini (1999)). Some studies consider that capacity drop may be caused by microscopic phenomena, such as lane-changes or heterogeneity among vehicles or lanes. On the other hand, bounded acceleration (BA) has also been considered to be responsible of capacity drop, Hall and Agyemang-Duah (1991), Khnoshyaran and Lebacque (2015), Jin (2018). The implementation of BA is motivated

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because many first order models lead to infinite acceleration and deceleration rates, e.g. the LWR-model (Lighthill and Whitham (1955), Richards (1956)). However, since the two-phase continuum model, Lebacque (1997), Lebacque (2003), very few continuum traffic flow models explicitly incorporate BA. The comparison between two recent studies, Jin (2017a) and Jin (2017b), shows that BA is a necessary and sufficient condition to reproduce capacity drop at lane-drop bottlenecks. This methodology has been expanded in Jin (2018) for sag and tunnel bottlenecks to reproduce the effects of capacity drop.

Aiming to increase the system's discharge-flow rate at bottlenecks, several control strategies have been proposed in the literature, e.g. ramp metering and variable speed limits (VSL). In this context, VSL aim to reduce the traffic flow entering a critical bottleneck by lowering the speed of the vehicles (i.e. substantially lower than critical speed), so as to prevent, eliminate, or delay the effects of capacity drop. The main drawback is the queue formation and consequent shock wave that propagates upstream, which might block off-ramps and worsen the traffic efficiency due to queue spill-back Carlson et al. (2010b). Therefore, it can be inferred that an adequate location of the VSL application area is crutial to improve traffic performance. Moreover, a recent analytical and numerical study on lane-drop bottlenecks Martínez and Jin (2018) argues that an optimal location for the VSL application area exists. Previous studies such as Carlson et al. (2010a) and Chen et al. (2014) assumed that a distance is needed between the end of the VSL application and the bottleneck, to allow vehicles to accelerate to a certain speed (usually the critical speed) before entering the bottleneck. Nevertheless, in Martínez and Jin (2018) it was shown mathematically that: (i) there exists a minimum distance between the VSL application area and the bottleneck in order to prevent the occurrence of capacity drop, even though this minimum distance could be zero under certain conditions; (ii) vehicles do not need to accelerate to uncongested states before entering the bottleneck to avoid capacity drop; and (iii) the larger speed limit, the longer distance is needed. The present work aims to extend the results obtained in Martínez and Jin (2018) to other types of bottlenecks, specifically to sag and tunnel bottlenecks. Sags are freeway stretches along which the gradient changes significantly (increasing with the direction of flow). In several studies it is discussed how drivers change their driving behavior on these uphill sections.

The rest of the paper is structured as follows. In Section 2 the bottleneck, the bounded acceleration and the continuum car following model are presented. In Section 3 the mathematical formulation in Martínez and Jin (2018) is extended for sag and tunnel bottlenecks and a theorem is presended to simplify the optimization problem. Moreover, in Section 4 numerical simulations with a continuum car-following model are used to analyze the impacts of the control location on capacity drop for the Kobotonoke tunnel (Koshi et al. (1992)). Finally, we conclude the study in Section 5 with a discussion and future directions.

2. Model

2.1. Sag or tunnel bottleneck definition

Two phenomena are related to capacity drop in sags Koshi et al. (1992), Koshi (2003): (i) a particular change in car-following behavior and (ii) extreamly low accelerations downstream of the bottleneck. Based on Goñi Ros et al. (2013) drivers inside the bottleneck keep larger distance to the leader than on the downhill section for similar speeds. On the other hand, in a tunnel, a driver tends to drive more carefully due to the low-light conditions and also leaves a larger distance. In summary, the capacity reduction in these type of bottlenecks can be explained by a time gap increase, which equals the time for a follower to cover the distance (clearance) to the front vehicle. Assuming increasing time gaps, a location-dependent triangular fundamental diagram Munjal et al. (1971) can be expressed as in (1). The authors assume that the free-flow speed and the jam density are constant at all locations. The bottleneck is defined by increasing linearly the time gap between $x = 0$ and $x = L$ (2).

$$q(x,t) = \min\left\{v_f k(x,t); \frac{1}{\tau(x)}\left(1 - \frac{k(x,t)}{k_j}\right)\right\} \tag{1}$$

$$\tau(x) = \begin{cases} \tau_1 & \text{if } x < 0 \\ \tau_1 + \dfrac{\tau_2 - \tau_1}{L}x & \text{if } 0 < x < L \\ \tau_2 & \text{if } x > L \end{cases} \tag{2}$$

2.2. Bounded acceleration models

Several BA criteria have been proposed in the literature. The constant bounded acceleration (CBA) with a_0 is the simplest one, other models include a speed-acceleration relationship, e.g. TWOPAS model Allen et al. (2000) (3) or Gipps model, Gipps (1981). In Jin (2018) it was proven that the Riemann problem with entropy condition has a unique solution when the constraint for bounded acceleration is introduced inside the bottleneck. This bounded acceleration model needs to fulfill several criteria: (i) be non-negative $A(x, v) \geq 0$, (ii) be bounded by a maximum acceleration rate, $A(x, v) \leq a_0$; and (iii) have a non-increasing relation with speed, i.e. $\frac{\partial A}{\partial v} \leq 0$.

$$A_{max}(v) = (a_0 - g\Phi(x))\left(1 - \frac{v}{v_f}\right) \tag{3}$$

2.3. Continuum car-following

The second order model used is an extension of the kinematic wave model developed in Jin (2018), considering the equivalences in Jin (2016). It considers both vehicles and time continuum variables, which are discretized (4). This model is able to reproduce the capacity drop in an endogenous way when upstream demand exceeds the capacity of the downstream road. The dropped capacity depends on the fundamental diagram definition and the BA-model.

$$\begin{cases} X_t(t + \Delta t, N) = \min\left\{X_t(t, N) + \Delta t \cdot A_{max}(X_t(t, N)); V\left(\frac{X(t, N - \Delta N) - X(t, N)}{\Delta N}\right)\right\} \\ X(t + \Delta t, N) = X(t, N) + X_t(t + \Delta t, N) \cdot \Delta t \end{cases} \tag{4}$$

3. The control impacts

3.1. Speed limit

The interest of the present work is to study the impacts of control location on traffic. Consequently, an open-loop control is designed, i.e. time-independent. Its goal is to control the demand at the bottleneck section. Therefore, it should be able to accommodate the downstream supply.

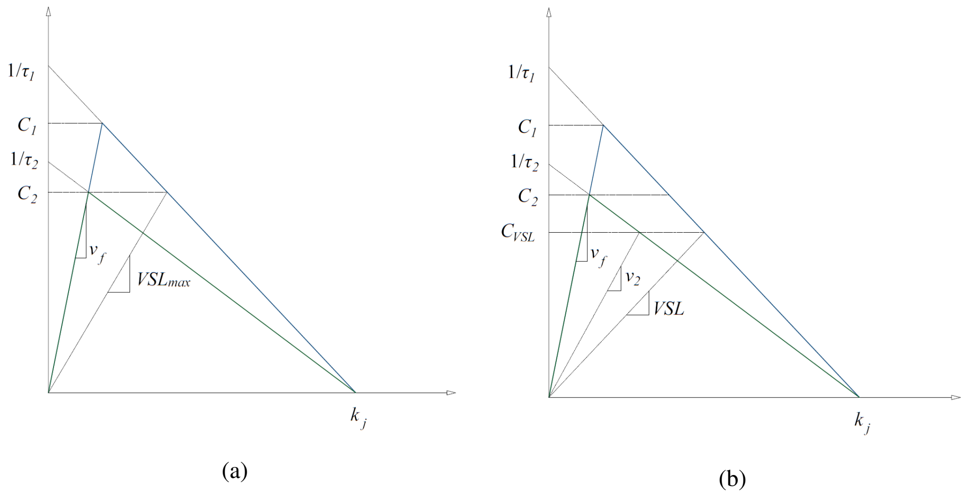


Fig. 1: (a) Maximum speed limit, VSL_{max}, defined by downstream capacity, C_2. (b) Lower speed limit and its associated flow rate, C_{VSL}, and speed for congested downstream states, v_2.

Assuming that the speed-density relation of drivers is not affected by the control implementation, the election of such speed limit can be deduced from Figure 1. It is evident that there is a maximum VSL (5), i.e. the speed that allows a throughput as high as the downstream capacity, C_2. Lower speed limits are also feasible, however they will further reduce the flow rate exiting the control. The maximum flow observed for a given speed limit, C_{VSL}, can be calculated from (6).

$$VSL_{max} = \frac{v_f}{v_f k_j (\tau_2 - \tau_1) + 1} \tag{5}$$

$$C_{VSL} = \frac{k_j VSL}{k_j \tau_1 VSL + 1} = \left(\tau_1 + \frac{1}{k_j VSL}\right)^{-1} \tag{6}$$

3.2. Location of application area

To prevent capacity drop, the speed of vehicles exiting the controlled area must be determined by the bounded acceleration and not by the LWR-stationary state, Martínez and Jin (2018). Under these conditions the traffic states inside and downstream from the control application area are stationary. In that study it was argued that only for a certain control locations the equilibrium states inside the bottleneck could be driven to a BA-traffic states. A complicated mathematical formulation was derived to obtain the minimum distance between control end and the start of a lane-drop bottleneck. This formulation is extended here for a sag and tunnel bottlenecks, (7). Moreover, through Theorem 1, this mathematical problem can be simplified to a maximization problem with an ODE and two boundary conditions as constraints (8).

$$\begin{aligned}
&L_u^{opt} = \min_{L_u} |L_u| \\
&\text{s.t.} \\
&\frac{dv_*(x)}{dx} = \frac{A(v_*(x))}{v_*(x)} \\
&v_*(L_u) = VSL \\
&v_*(x) \geq \Psi(x) = \frac{C_{VSL}}{k_j - k_j \tau(x) C_{VSL}}, \forall x \in [L_u, L]
\end{aligned} \tag{7}$$

Theorem 1. *The speed is determined by the bounded acceleration for any location inside the bottleneck if and only if the speed at the entrance of the bottleneck is at least the speed imposed by the control, i.e.* $v_*(x = 0) \geq VSL$, *and the speed at the end of the bottleneck is at least the speed of the congested branch of the downstream fundamental diagram associated to the throughput that the control defines, i.e.* $v_*(x = L) \geq v_2$.

Proof. If $A(v_*(x))$ is a decreasing (or constant) function in speed and the speed $v_*(x)$ is increasing in x, the slope of $v_*(x)$ is decreasing in x, i.e. the speed profile is convex. Moreover, the boundary condition $v(L_u) = VSL$ determines the location where speed starts increasing.

On the other hand, the minimum speed at each location, $\Psi(x)$, (to ensure that vehicles are accelerating at maximum rate) is also strictly increasing with x. However, it increases in a hyperbolic shape. This can be proven because $\tau(x)$ is decreasing linearly with x. Thereafter, the slope of $\Psi(x)$ is strictly increasing inside the bottleneck and is constant for upper and downstream links. See Figure 2.

Consequently, the condition to ensure a BA traffic state inside the bottleneck ($v_*(x) \geq \Psi(x)$) is met if and only if the condition holds both at the start and end of the bottleneck, i.e. $v_*(0) \geq VSL$ and $v_*(L) \geq \Psi(L) = \dfrac{VSL}{k_j VSL(\tau_2 - \tau_1) + 1}$. Note that the condition at $x = 0$ is fulfilled automatically if the control location ends before (or at) the start of the bottleneck bottleneck (i.e. $L_u \leq 0$). □

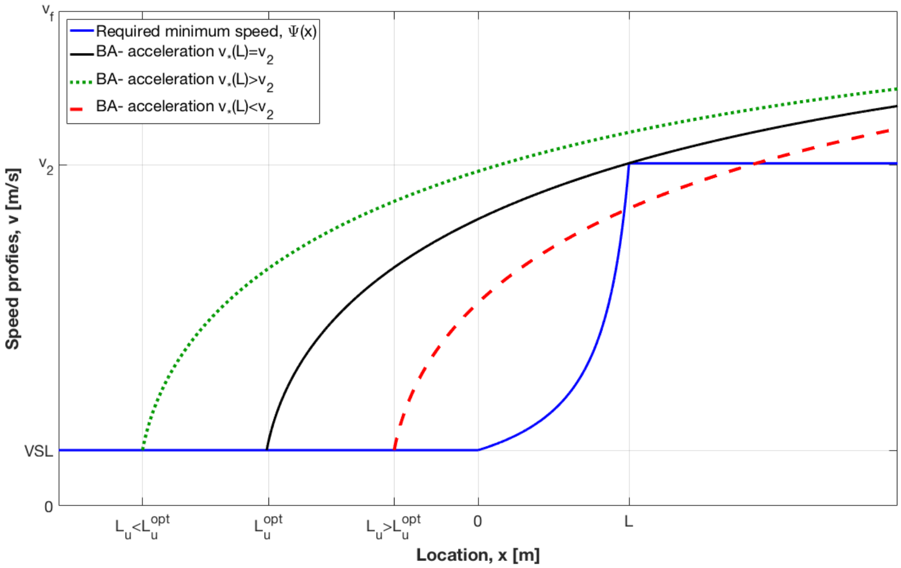


Fig. 2: Illustration on required speed at each location. The dashed-red speed profile is unfeasible, while dotted-green is feasible but not optimal. Only when $v_*(L) = v_2$ the control was implemented at the optimal application area, i.e. black speed profile.

The required distance between the end of the control application area and the end of the bottleneck to prevent capacity drop is called *critical length*. From Theorem 1, it can be concluded that the minimum theoretical critical length is the zone where the time gap is increasing, i.e. the bottleneck length. Depending on the downstream congested speed, v_2, and the bounded acceleration model, the critical length may be longer than the minimum critical length. Moreover, (8) can be solved analytically if the bounded acceleration is a constant. Then, the critical length is the distance required to accelerate from VSL to v_2, that can be obtained from basic kinematic equations. Otherwise a numerical solution is needed to solve the ODE in (8). This can be solved with an explicit scheme from $x = L$ integrating in the upstream direction.

$$\begin{aligned} &\max_{L_u} L_u \\ &\text{s.t.} \quad v(x)\frac{\mathrm{d}v(x)}{\mathrm{d}x} &&= A(v(x)) \\ &\qquad v(L_u) &&= VSL \\ &\qquad v(L) &&\geq v_2 = \frac{VSL}{k_j VSL(\tau_1 - \tau_2) + 1} \\ &\qquad L_u &&\leq 0 \end{aligned} \tag{8}$$

4. Numerical results

4.1. Without control

Considering Kobotonoke Tunnel bottleneck geometry (Koshi et al. (1992)), the model is applied assuming constant demand, the parameters used are obtained from Jin (2018) and are summarized in Table 1. The different time gaps along the freeway stretch are defined in (2). However, in this simulation the upgrade is not considered, because the maximum acceleration would be both location- and speed-dependent. In that case, Theorem 1 should be modified slightly and (8) may need to be revised if the acceleration is not strictly decreasing with x or non-monotone. Thereafter, the BA-model in (3) is considered with $\phi = 0$. In Figure 3 the simulations results without control are presented.

Table 1: Parameters of the simulation.

Parameter	Value	Units
Free-flow speed, v_f	80	km/h
Jam density, k_j	140	veh/km
Acceleration rate, a_0	0.407	m/s^2
Bottleneck length, L	1.5	km
Time gap upstream to the bottleneck, τ_1	1.5	s/veh
Highest time gap inside the bottleneck, τ_2	2.1	s/veh

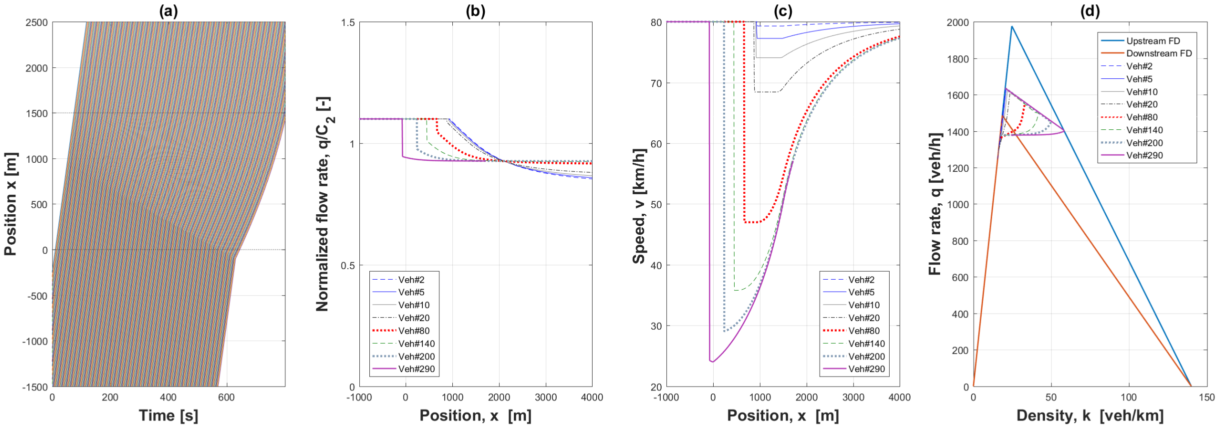


Fig. 3: (a) Trajectories of 300 vehicles. (b) Normalized flow rate (over downstream capacity) for several vehicles. (c) Speed profile of different vehicles. (d) Temporal evolution of traffic states for different vehicles.

4.2. With VSL control

By imposing the control of VSL, we aim to modify the equilibrium state of LWR model to obtain a BA-stationary state inside the bottleneck. It is important to note that since the BA model considered is TWOPAS, the control speed cannot be VSL_{max}. When the maximum speed is imposed in a control, v_2, is precisely the free-flow speed and this is never achieved with a TWOPAS BA model, due to its asymptotic shape. Consequently, the required L_u would be $x = -\infty$. To avoid this non-meaningful solution, the speed limit from (5) is slightly rounded down to the first decimal, i.e. $VSL = 27.5km/h$.

The numerical solution of (8) is presented in Figure 4. The speed profile is obtained by explicit Euler integration from the boundary condition at $x = L$. The obtained L_u^{opt} is $-1127m$. The simulation results in Figure 5 show how the adequate location of the control end is crucial for capacity drop prevention. It is important to highlight that the critical distance obtained through the simulation results is slightly higher than the obtained through numerical integration of (8). However, the difference is less than $45m$, which represents less than a 4% difference. This difference could be related to numerical errors.

5. Discussion

In this paper it has been proven that the definition of optimal VSL application area defined in Martínez and Jin (2018) can be extended to sag and tunnel bottlenecks. The numerical results presented in Section 4 show that if the BA stationary state is stable inside the bottleneck, capacity drop can be prevented. However, if the end of the control application is too close to the bottleneck, a sudden break at the exit of the bottleneck disturbs the BA stationary state. After a long enough period, the traffic states will be the stationary states observed without control, Figure 3.

Moreover, a simplification of the optimal location formulation in Martínez and Jin (2018) has been presented. Herewith, this location can be calculated analytically for more complicated BA-models than the constant BA. It has

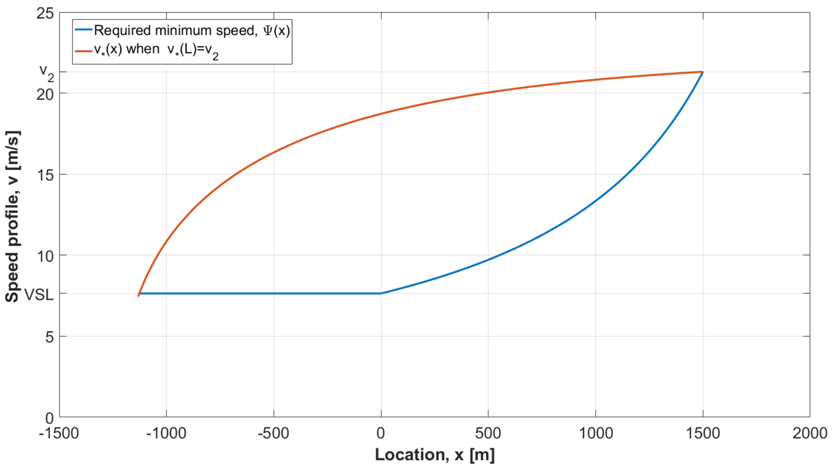
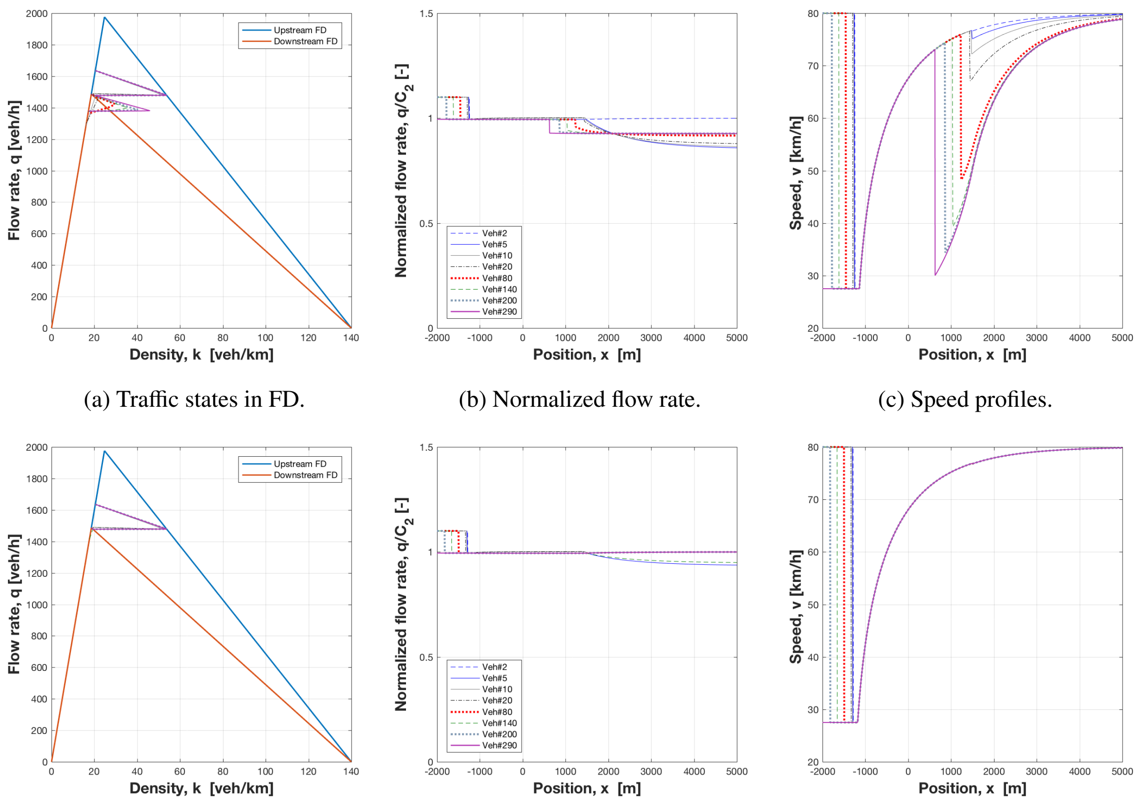


Fig. 4: Numerical calculation of speed profile under optimal conditions.



(a) Traffic states in FD.

(b) Normalized flow rate.

(c) Speed profiles.

(d) Traffic states in FD.

(e) Normalized flow rate.

(f) Speed profiles.

Fig. 5: (a)-(c) Control at $L_u = -1140$. After several vehicles break down the acceleration process inside the bottleneck is defined by LWR states and not bounded by the BA model. Capacity drop appears. (d)-(f) Control at $L_u = -1170$. Stationary speed profiles, acceleration of vehicles is bounded by BA model at all locations. Capacity drop prevention.

also been discussed that in TWOPAS model (as well as in Gipps model) v_f is never achieved. Thus, the speed profile, $v_*(x)$, will have an horizontal asymptote and the theoretical maximum speed limit VLS_{max} cannot be implemented. However, real vehicles do accelerate to free-flow speed. This suggests that other more realistic BA-models should be defined in the future. As long as the BA meets the conditions to generate a convex speed profile, Theorem 1 can be used to simplify the optimal mathematical formulation. In other words, the methodology presented can be applied for any acceleration-speed relation, $A_{max}(x, v)$, that is non-negative and non-increasing with speed.

It is important to highlight that the bottleneck defined in this paper considered reduced downstream capacity on the whole link (2). However, when the tunnel ends, vehicles might be able to reduce their time gaps again to reach τ_1. Modifications on the bottleneck geometry are not expected to modify the minimum required speed $\Psi(x)$. However, the last condition on (8) implicitly assumes that the associated LWR-equilibrium traffic state is in the congested branch of the fundamental diagram. Thereafter, the formulation of optimal location should be revised and maybe extended to ensure the stability of the bounded acceleration process from $v_*(L) = v_2$ to free-flow speed, when the downstream capacity is higher than C_2. Moreover, a sag bottleneck with piece-wise definitions of slope inside and downstream of the bottleneck might also influence the stability of the bounded acceleration stationary states. In conclusion, further research on the effects of downstream road characteristics are needed.

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