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Optimization of Urban Transit Network Design and Timetabling for Round-trip Routes

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Abstract

A MIP model of solving planning problem of network design and timetabling for urban bus systems simultaneously is developed. Mathematical formulation for solving TNDTP is usually an intractable problem, because it combines three classes planning activities of TNP into a single problem. Therefore, the TNDTP is used to be modified as a size-limited and simplified problem in the previous research (Guihaire and Hao, 2008). Integrated planning of network design and timetabling for urban bus systems simultaneously has not been developed in studies before. There are two main contributions of this research. One is a mixed-integer (linear) programming model that describes urban TNDTP with multi-depot and round-trip routes is proposed first time, which is the principle achievement. The other contribution is the proposed model adopts a multi-objectives which consider both of operator and user sides. The model also accommodates unsatisfied demand, bus depot, route pattern, route length, frequency bound, headway structure, bus capacity, and transit assignment in a single problem. It has never been considered before. Then, a computational study is conducted to evaluate the performance of the proposed model and to judge the effects of parameters on the results. Finally, the outcomes and statistics of operators and passengers are reasonable. This shows that the proposed methodology is useful for simultaneously planning bus network design and timetabling.

Keywords: network design; urban public transit; timetable; mixed-integer programming

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1. Introduction

There are three main planning problems–network design, frequency setting, and timetabling–included in the transit network problem (TNP), which solves the design of a transit system. (Guihaire and Hao, 2008). Besides, the integration of all the planning activities is called TNDTP problem. However, formulating and solving process of TNDTP are bulky, so the TNDTP considered in the literature is usually size-limited and highly simplified (Guihaire and Hao, 2008). There are some models developed in related studies are solved sequentially insteand of a single problem, like the two-paper series of Shrivastav and Dhingra (2001) and Shrivastava and Dhingra (2002). For ease of problem solving, many algorithms have been already used in the literature. For example, Yan et al. (2006) and Yan and Tang (2008) adopted heuristics algorithms to address the problem; Yan and Chen (2002) used Lagrangian relaxation for solving. In addition, some stochastic factors have been considered in the past studies. Yan et al. (2006) and Yan and Tang (2008) take stochastic demand and stochastic travel time into consideration respectively, and essentially belonged to a different class of problems. The preceding discussion that what kind of algorithms is adopted shows the high complexity of TNDTP indeed. However, the exact solution algorithms have not yet been successfully developed for TNDTP.

This study proposes an innovative model for urban TNDTP that concentrates on urban bus systems with multi-depot round-trip routes. Furthermore, it is the first to develop a mixed-integer (linear) programming model for such a problem. What's more, to solve the aforementioned problem, a parallel branch-and-price-and-cut (BPC) algorithm is developed. This algorithm is the first exact solution approach for the problem. The multi-objectives of proposed model for operator and passenger consider unsatisfied demand, bus depot, route pattern, route length, frequency bound, headway structure, bus capacity, fleet size, and transit assignment together.

2. Methodology

2.1. Problem statement

Given a graph symbolizing the road network. A sequence of connected network nodes represents a bus *route*. Additionally, the bus routes are round-trip and restricted by the maximum route length. A bus *dispatch pattern* is defined as a collection of times during which buses are dispatched from the depot. There are two types of dispatch patterns. One is constant headway, and the other is variable headway. In either type, the headways of a dispatch pattern must follow the requirements for minimum headway (or equivalently maximum frequency). The route and dispatch pattern forms a *timetable*. A bus *line* is a combination of a bus route, a dispatch pattern, and a timetable, and the outcome must satisfy the requirements for the fleet size of each bus line. Minimize the weighted sum of operator cost, passenger generalized cost, and penalty for unsatisfied demand is the objective of solving problem. Time-dependent OD pairs is grouped passenger demands. If a path between an OD pair satisfies given criteria, like generalized travel cost, total travel time, total waiting time, and total number of transfers, it is called *feasible*.

The major assumptions for the model are shown below:

1. Each road is two-way streets. Straightforwardly, one-way street can't have a round-trip route. Thus, if there is any one-way streets exist, then other types of routes must be considered. All roads in the problem are set to be two-way roads to focus on round-trip routes. In the real world, if a one-way streets do exist, the problem can be solved by first assuming that all streets are two-way. Then, the bus routes and timetables can be modified to adapt to the one-way streets later.
2. Travel times of each link are known and deterministic in advance.
3. Bus can stop at all network nodes for boarding and alighting. Moreover, the time of boarding and alighting both of are constant and counted in the travel time of link.

2.2. Model formulation

Given a discrete-time time-expanded network and the terminologies of *physical* network/links/nodes and *time-expanded* network/links/nodes are used. The notations for the model are defined as follows.

Sets:

- N: set of physical network nodes, indexed by i, j, and k.
- D: set of depots, indexed by d, $D \subseteq N$.
- A: set of physical network links, indexed by a tuple of two nodes.
- L: set of candidate bus lines, indexed by l.
- G: set of all origin-destination pairs, indexed by g.
- P_g: set of passenger paths of OD pair $g \in G$, indexed by p.

Parameters:

- T: number of periods.
- T^d: number of periods that a bus can be dispatched from the depot.
- w^b,w^g,w^p: weights for bus-operating cost, passenger generalized cost, and penalty for unsatisfied demand, where $w^b + w^g + w^p = 1$.
- α^1_{ij}: bus-operating cost from node i to node j.
- α^2_{gp}: generalized cost for passenger path p of OD pair g.
- α^3_g: penalty for each unsatisfied passenger of OD pair g.
- q_g: travel demand of OD pair g.
- d_l: depot for line l.
- $dist_{ij}$: travel distance from node i to node j.
- $time_{ijt}$: travel time from node i to node j departing at time t. The travel times are restricted to integers.
- max^d: maximum travel distance allowed for a bus route.
- min^h, max^f: minimum headway and corresponding (maximum) frequency for a bus line, where $max^f = \lceil \frac{T^d}{min^h} \rceil$.
- max^t: maximum travel time allowed for a bus route.
- k: capacity of a bus.
- δ_{gpijt}: indicator parameter that shows whether passenger path p of OD pair g passes link (i, j) at time t.
- f: fleet size of a bus line.

Integer variables:

- X_{ijl}: route variables, 1 if route for line l traverses link (i,j) and 0 otherwise.
- $\tilde{Y}_{ij\tilde{t}l}$: "template" timetable variables, 1 if the template timetable of line l traverses link (i,j) departing at time $\tilde{t}$ and 0 otherwise.
- Y_{ijtl}: timetable variables, 1 if the timetable of line l traverses link (i,j) departing at time t and 0 otherwise.
- Z_{tl}: dispatch variables, 1 if a bus is dispatched from depot at time t for line l and 0 otherwise.
- $\Delta_{ijt\tilde{t}l}$: auxiliary variables used in Eqs. (32)-(35). 1 if a bus passing link (i, j) exists at time $\tilde{t}$ in the template timetable and a bus is dispatched at time $t - (\tilde{t} - 1)$ and 0 otherwise.
- R_{il}: route node variables, 1 if node i is on bus route l and 0 otherwise.
- I_{il}: intermediate node variables, 1 if node i is an intermediate node of bus route l and 0 otherwise.
- H_l: constant headway of bus line l.
- F_l: dispatch time of first bus for bus line l.

Continuous variables:

- E_{tl}: time elapsed since the previous dispatch for bus line l at time t.

- U_g: portion of demand for OD pair g that is unsatisfied.
- W_{gp}: portion of demand for OD pair g that takes passenger path p.

The objective function of the model is given in Eq. (1). It is to minimize the weighted sum of bus operating cost, passenger generalized cost, and penalty for unsatisfied demand.

$$\min w^b \sum_{(i,j)\in A} \sum_{t=1}^{T} \sum_{l\in L} \alpha_{ij}^1 Y_{ijtl} + w^g \sum_{g\in G} \sum_{p\in P_g} \alpha_{gp}^2 q_g W_{gp} + w^p \sum_{g\in G} \alpha_g^3 q_g U_g \tag{1}$$

Types of constraints and corresponding description are as follows:

- Route: the constraints of route ensure that the route variables will form a round-trip route and state that each route must leave its depot exactly once.

$$\sum_{(d_l,j)\in A} X_{d_l jl} = 1, \forall l \in L \tag{2}$$

$$\sum_{(i,j)\in A} X_{ijl} \leq 2, \forall i \in N, i \neq d_l, l \in L \tag{3}$$

$$X_{ijl} = X_{jil}, \forall (i,j) \in A, l \in L \tag{4}$$

$$\sum_{(i,j)\in A} X_{ijl} = \sum_{(j,k)\in A} X_{jkl}, \forall j \in N, l \in L \tag{5}$$

$$\sum_{(i,j)\in A} dist_{ij} X_{ijl} \leq max^d, \forall l \in L \tag{6}$$

- Headway: the dispatch variables of a bus line are controlled by the headway constraints. What's more, both of variable and constant headway are considered.

  Variable headways.

$$\sum_{t=1}^{T^d} Z_{tl} \leq max^f, \forall l \in L \tag{7}$$

  Constant headways.

$$H_l \geq min^h, \forall l \in L \tag{8}$$

$$H_l \leq T^d + 1, \forall l \in L \tag{9}$$

$$F_l \leq H_l, \forall l \in L \tag{10}$$

$$E_{1l} = H_l - F_l + 1, \forall l \in L \tag{11}$$

$$E_{t+1,l} \geq 1 - T^d(1 - Z_{tl}), \forall t \in \{1, \cdots, T^d - 1\}, l \in L \tag{12}$$

$$E_{t+1,l} \leq 1 + T^d(1 - Z_{tl}), \forall t \in \{1, \cdots, T^d - 1\}, l \in L \tag{13}$$

$$E_{t+1,l} \leq (E_{tl} + 1) + T^d Z_{tl}, \forall t \in \{1, \cdots, T^d - 1\}, l \in L \tag{14}$$

$$E_{t+1,l} \geq (E_{tl} + 1) - T^d Z_{tl}, \forall t \in \{1, \cdots, T^d - 1\}, l \in L \tag{15}$$

$$E_{tl} \geq H_l + (-T^d)(1 - Z_{tl}), \forall t \in \{1, \cdots, T^d\}, l \in L \tag{16}$$

$$E_{tl} \leq H_l + (T^d + 1)Z_{tl} - 1, \forall t \in \{1, \cdots, T^d\}, l \in L \tag{17}$$

- Timetable: the timetable variables are regulated by the timetabling constraints and guarantee that the bus timetable is formed in correspondence to the route and the dispatch pattern.

Dynamic travel times.

$$Y_{ijtl} \leq X_{ijl}, \forall (i,j) \in A, t \in \{1, \cdots, T\}, l \in L \tag{18}$$

$$\sum_{t=1}^{T} Y_{ijtl} \leq \sum_{t=1}^{T^d} Z_{tl}, \forall (i,j) \in A, l \in L \tag{19}$$

$$\sum_{t=1}^{T} Y_{ijtl} \geq \sum_{t=1}^{T^d} Z_{tl} - max^f (1 - X_{ijl}), \forall (i,j) \in A, l \in L \tag{20}$$

$$X_{d_l jl} + Z_{tl} \leq Y_{d_l jtl} + 1, \forall t \in \{1, \cdots, T^d\}, l \in L, (d_l, j) \in A. \tag{21}$$

$$\sum_{\{(i,j)\in A, t'\in\{1,\cdots,t-1\} | t'+time_{ijt'}=t\}} Y_{ijt'l} = \sum_{(j,k)\in A} Y_{jktl}, \forall j \neq d_l, t \in \{1, \cdots, T\}, l \in L \tag{22}$$

$$R_{il} \leq \sum_{(i,j)\in A} X_{ijl}, \forall i \in N, l \in L \tag{23}$$

$$2R_{il} \geq \sum_{(i,j)\in A} X_{ijl}, \forall i \in N, l \in L \tag{24}$$

$$2I_{il} \leq \sum_{(i,j)\in A} X_{ijl}, \forall i \in N, l \in L \tag{25}$$

$$I_{il} \geq \sum_{(i,j)\in A} X_{ijl} - 1, \forall i \in N, l \in L \tag{26}$$

$$\begin{aligned} I_{jl} + X_{ijl} + Y_{ijt'l} + X_{jkl} &\leq Y_{jktl} + 3 \\ &\forall j \in N, j \neq d_l, (i,j) \in A, (j,k) \in A, i \neq k, \\ &t \in \{1, \cdots, T\}, t' \in \{1, \cdots, t-1\}, t' + time_{ijt'} = t, l \in L \end{aligned} \tag{27}$$

$$\begin{aligned} R_{jl} + (1 - I_{jl}) + Y_{ijt'l} &\leq Y_{jitl} + 2 \\ &\forall j \in N, j \neq d_l, (i,j) \in A, \\ &t \in \{1, \cdots, T\}, t' \in \{1, \cdots, t-1\}, t' + time_{ijt'} = t, l \in L \end{aligned} \tag{28}$$

Static travel times. A template timetable has two requirements. One is the template timetable should be consistent with the route and link travel times. Another requirement is it must be connected.

$$\sum_{(d_l,j)\in A} \tilde{Y}_{ij1l} = 1, \forall l \in L \tag{29}$$

$$\sum_{\tilde{t}=1}^{max^t} \tilde{Y}_{ij\tilde{t}l} = X_{ijl}, \forall (i,j) \in A, \forall l \in L \tag{30}$$

$$\sum_{\{(i,j)\in A | \tilde{t}-time_{ij\tilde{t}}\geq 1\}} \tilde{Y}_{ij\tilde{t}l} = \sum_{(j,k)\in A} \tilde{Y}_{jk\tilde{t}l}, \forall j \neq d_l, \tilde{t} \in \{1, \cdots, max^t\}, l \in L \tag{31}$$

$$\begin{aligned} Z_{t-(\tilde{t}-1),l} &\geq \Delta_{ijt\tilde{t}l}, \\ &\forall (i,j) \in A, t \in \{1, \cdots, T\}, \tilde{t} \in \{1, \cdots, max^t\}, t - (\tilde{t} - 1) \geq 1, l \in L \end{aligned} \tag{32}$$

$$\begin{aligned} \tilde{Y}_{ij\tilde{t}l} &\geq \Delta_{ijt\tilde{t}l}, \\ &\forall (i,j) \in A, t \in \{1, \cdots, T\}, \tilde{t} \in \{1, \cdots, max^t\}, t - (\tilde{t} - 1) \geq 1, l \in L \end{aligned} \tag{33}$$

$$Z_{t-(\tilde{t}-1),l} + \tilde{Y}_{ij\tilde{t}l} \leq \Delta_{ijt\tilde{t}l} + 1,$$

$$\forall (i,j) \in A, t \in \{1,\cdots,T\}, \tilde{t} \in \{1,\cdots,max^t\}, t-(\tilde{t}-1) \geq 1, l \in L \tag{34}$$

$$Y_{ijtl} = \sum_{\tilde{t} \in \{1,\cdots,max^t | t-(\tilde{t}-1) \geq 1\}} \Delta_{ijt\tilde{t}l}, \forall (i,j) \in A, t \in \{1,\cdots,T\}, l \in L \tag{35}$$

- Fleet size: the constraints regulate the total number of running buses of a line will not more than the given fleet size at any time period.

$$\sum_{\{(i,j) \in A, t' \in \{1,\cdots,t\} | t' + time_{ijt'} = t\}} Y_{ijt'l} \leq f, \forall t \in \{1,\cdots,T\}, l \in L \tag{36}$$

2.3. Passenger path generation

An OD pair are assumed to share only a common departure time at the origin but not an arrival time at the destination based on the breadth-first search (BFS). A search tree is maintained to record the search progress storing all currently found paths. A branch of the search tree can be bounded to eliminate the search space under two situations. First, the branch is bounded when the destination is reached (at any period), because of finding a feasible path. Second, the branch is also bounded when any of the conditions for infeasible paths is satisfied. The reason why is further search along the branch will not give any feasible path. The conditions for infeasible passenger paths considered in this study are shown below:

1. Travel time larger than the maximum total travel time.
2. The number of waiting transfer surpasses the maximum number of total waiting transfer.
3. Transfer waiting time exceeds the maximum transfer waiting time.
4. Initial waiting time exceeds the maximum initial waiting time.
5. At the end of the planning horizon, passengers still cannot reach their destination.

3. Parallel BPC method

A parallel BPC algorithm is developed to solve the proposed TNDTP for urban bus systems. The algorithm applies to both static and dynamic travel times. Besides, BPC is an integration of branch-and-price and branch-and-cut algorithms. With linear programming (LP) relaxations, the branch-and-price algorithm is a variant of branch-and-bound (B&B). The branch-and-price algorithm is using column generation to solve the LP relaxation in each node of a B&B tree. When no columns price out and the integrality conditions are not satisfied, branching is required. The branch-and-cut algorithm is another variant of B&B with LP relaxation.

4. Computational Results

The performance of the proposed BPC algorithm is demonstrated by conducted computational study. The parallel BPC algorithm is written in Python programming language. Moreover, the solver of LP and MIP is Gurobi 7.0. All the problem instances of the example are solved using a single desktop PC with Intel Core i7 3.40 GHz CPU with 32 GB RAM. The test cases of computational study are modified from the classical example in Mandl (1979).

The basic settings for the computational study and the parameters for the operator are summarized.

- The operating speed of each bus is 40 kph. The travel distance for each link is estimated corresponding its travel time and assumed speed.
- Considering the structure of the network, the maximum travel distance for a route is estimated via a round-trip route across the network. The route 0-1-3-5-7-9-13-9-7-5-3-1-0 is used, and its travel distance is 46. The corresponding travel time is 72 min (24 periods).
- To allow buses to return to the depot by the end of the planning horizon, no bus is dispatched in the last 24 periods (the maximum travel time for a route) to allow buses.

- The bus operating cost is NT$ 54.6 per km.
- The capacity of each bus is 50 persons.
- The fleet size is 24.
- Ticket price is not considered, i.e., $\alpha_4 = 0$.

The parameters for the passengers are as follows.

- The assumption that the daily demand is distributed uniformly within a 12-h period or 240 time periods is established, Time periods contains 11910 time-dependent OD pairs.
- The first 18 min (6 periods) is reserved for bus operations, and no passenger demand occurs during this duration, thereby allowing the operator to serve early passengers whose origins are far from any depot.
- The maximum waiting time (initial and transfer combined) is set to 9 min (3 periods).
- The maximum number of waiting transfer is 1.
- The cost for in-vehicle time and out-of-vehicle time is estimated based on the value of time of bus users. The in-vehicle time cost is set to NT$ 1.75 per minute, and the out-of-vehicle time (including initial and transfer waiting times) is set to NT$ 2.47 per minute (Chen and Lin, 2009). The generalized cost for each feasible path is calculated using these values.
- The penalty for the unsatisfied demand of an OD pair (α_g^3) is set to 1.05 times of the largest generalized costs for all passenger paths of that OD pair.
- The first type of maximum travel time for an OD pair is double the shortest possible travel time between the OD pair.

The results are presented in Tables 1. When using MIP (constant/static) in Case 1, after 72 h of solution time, the gap between the lower and upper bounds of the optimal value was 50.1%. Then, the formulation of MIP with dynamic link travel times are tested (although the travel times in Case 1 do not change over time). After presolve, the MIP model has 80204 constraints and 186368 variables, in which 164892 are continuous and 21476 are integer (21460 binary). After 72 h, the gap is 59.7%. Because of the large problem scales, it shows using the MIP to solve the Case 1 is inefficient. Then, the performance of the BPC algorithm is tested. two sets of results are recorded. The first set is the results of the root node; the second one is the results of a total solution time of 4 h. The solution time for the root node contains two parts: time for column generation and integer solution. If column generation at the root node does not converge within 4 h, column generation is terminated and the binary programming for the integer solution is solved up to 1 h. Case 1 (BPC) indicates that using the proposed BPC algorithm to solve Case 1 and the optimality gap of it was

Table 1. Computational results

Case 1 (Constant-headway)

Solution approach	User cost weight	Penalty weight	Depot /line	Min headway (period)	Total time (period)	Total demand	Root node				Branching			
							LB	UB	Gap(%)	Time (s) *	LB	UB	Gap(%)	Time (s)
MIP (static)	0.500	0.500	4/8	2	70	2614					25063.5	50233.4	50.1	259200
MIP (dynamic)											25082.7	62306.2	59.7	259200
BPC							25569.9	26085.1	2.0	162+60	25655.2	26085.1	1.6	14191

* Solution time for column generation + solution time for integer solution.

only 2.0% after having solved the root node in 222 s. The gap is further reduced to 1.6% under a total solution time of 4 h. Comparing the statistics for the BPC approach and the MIP model, Case 1 (BPC) performs better in all statistics for passengers with longer bus routes and also has lower unsatisfied demand. However, the outcome represents that Case 1 (BPC) underestimates the transfer inconvenience providing a better solution. Overall, the proposed BPC algorithm is superior to using the off-the-shelf solver to solve the MIP formulations.

5. Discussion

A model for the simultaneous planning of route design and timetabling for urban bus systems with multi-depot round-trip routes is presented in this research. The essential notion of the model formulation is to depict a bus timetable with a route and a dispatch pattern. Furthermore, to solve the proposed model, a parallel BPC algorithm is developed. It is efficient indeed because it conducts in matrix form with existing linear algebra libraries. Eventually, the result of the routes and timetables as well as the statistics of operators and passengers show the rationality demonstrating the proposed model and the solution algorithm are useful tools.

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