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Modified Generalized Definitions for the Traffic Flow Characteristics under Heterogeneous, No-Lane Disciplined Traffic Streams

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Abstract

Efficient utilization of the transport infrastructure requires appropriate control and management strategies. Macroscopic traffic stream models are essential for developing such strategies. Modelling any physical system involves the identification of certain system variables which characterize its performances. Barring a few exceptions including the concept of area occupancy, progress in this direction is negligible. Present study attempts to characterize the heterogeneous, no lane-disciplined traffic stream by appropriately defining various measures of the traffic states. The authors have modified the Edie's generalized definitions by incorporating an additional dimension of the space. The proposed definitions are capable of capturing the vehicle heterogeneity as well as the no lane-disciplined driving. Advantages of the proposed definitions were tested over an hypothetical traffic stream as well as using the empirical data and found that the proposed definitions are useful in characterizing the heterogeneous traffic stream.

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Keywords: Traffic Flow Characteristics; Macroscopic Modelling; Area Occupancy; Heterogeneous Traffic; No Lane-Discipline; Edie's Generalized Definitions;

1. Introduction

By assuming the traffic stream as a fluid continuum, the traffic states can be represented with three fundamental characteristics, namely, the flow (quantity per unit time), density (quantity per unit space), and the speed (space per unit time) (Edie, 1963). The existence of the reproducible bi-variate relationships among the traffic flow characteristics is the basis for all the existing deterministic and dynamic traffic flow models. In developed countries, attempts to characterize and model the traffic stream date back to 1950's. Starting from Greenshields et al. (1935) to the present day researches, a remarkable progress has been made in characterizing the homogeneous, lane-disciplined traffic

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stream. The understanding obtained from these studies was translated into various management and control strategies necessary for optimal utilization of the transport infrastructure.

| Nomenclature | |
|--------------|---|
| q | Flow |
| k | Density |
| и | Space mean speed |
| W | Width of road under study |
| L | Length of road under study |
| Т | Time interval for which the parameters are averaged |
| q_A | Area flow |
| k_A | Area density |
| RFR | Road space freeing rate |

The traffic flow characteristics are defined in a number of ways with respect to the measurement methods (Wardrop, 1952; Edie, 1963). In a more general sense, these characteristics can be defined as follows. *Flow* is the temporal representation of the traffic states which measures the traffic intensity or the demand for the supplied infrastructure. It is expressed as the number of vehicles passing a stationary observer in an unit interval of time. *Density* is the spatial representation of the traffic states which measures the crowdedness of the traffic stream. It is measured as the number of vehicles seen over a unit road length for a time instance. Whereas the space mean speed is generally considered as the average traffic stream speed, which is defined as the harmonic mean of the spot speeds of vehicles found during the observation period. Space mean speed can also be defined as the arithematic average of the instantaneous speeds of vehicles over a length of roadway. The average speed measures the efficiency of the dynamics of the traffic stream. The fundamental relationship between these characteristics is as follows.

$Flow = Density \times S peed$

Identifying the appropriate macroscopic traffic flow characteristics and the relation between them is an integral part of the macroscopic traffic flow modelling. By definition, the macroscopic characteristics, the flow, speed, and the density, are applicable only for homogeneous and lane disciplined traffic streams. Furthermore, it is clear from the definitions that all these parameters are measured at different time-space frameworks. Hence the existence of the fundamental relationship between these parameters can be easily questioned. Cassidy (1998) found that a reproducible bi-variate relation exists between the traffic stream characteristics only if the measured data correspond to nearly stationary traffic conditions. As the fundamental relationship comprises both the spatial and temporal variables, it may be incompatible with one another in non-uniform traffic streams (Banks et al., 1995). Similarly, many researchers have questioned the consideration of speed as the ratio of flow and density since the averaging can be done in several ways (Hall and Persaud, 1989; Hall, 1996; Banks et al., 1995). The generalized definitions for the traffic stream characteristics proposed by Edie (1963) was capable of resolving all the above controversies (Cassidy and Coifman, 1997). Within the Edie's generalized definitions, the fundamental relationship between the traffic stream characteristics holds as an identity (i.e., it is true by definition) (Cassidy and Coifman, 1997). A detailed discussion over the Edie's definitions of fundamental traffic stream characteristics is provided in Section 2.

For most of the developing countries, the traffic stream comprises vehicles with varying physical and dynamical characteristics and the drivers do not follow the lane markings. The compatibility of the above definitions and the relationships among the traffic flow characteristics under heterogeneous and no lane-disciplined traffic conditions was questioned by several researchers (Lam and Huang, 1992; Khan and Maini, 1999; Oketch, 2000; Mallikarjuna and Rao, 2006a; Kiran and Verma, 2016). The existing practice of the macroscopic analysis of such traffic stream converts the traffic stream into an equivalent homogeneous traffic stream using the passenger car units (PCU). Several researchers have proposed different set of PCU values for the heterogeneous traffic stream (e.g., Mallikarjuna and Rao, 2006b; Arkatkar, 2018; Tiwari et al., 2000; Logghe and Immers, 2003; Basu et al., 2006; Krishnamurthy et al., 2008; Arasan and Arkatkar, 2010; Dhamaniya and Chandra, 2013). The adequacy of the PCU's and the existing

models in representing the heterogeneous traffic stream has achieved only limited success and has involved much re-calibration effort and a significant model modifications (Oketch, 2000; Lam and Huang, 1992). Hence, researchers have pointed out the importance of putting more research focus on defining appropriate macroscopic characteristics which are relevant for heterogeneous traffic stream (Mallikarjuna and Rao, 2006a; Khan and Maini, 1999). Some of the attempts on this aspect include the work of Mallikarjuna and Rao (2006a), who have proposed a more meaningful representation for the traffic concentration, in terms of the '*Area Occupancy*'. The concept of area occupancy was an alternate to the occupancy used for characterizing the homogeneous traffic stream. Mallikarjuna and Rao (2006a) have defined the area occupancy as the proportion of time for which the vehicle area occupies the detection zone. Present study extends the concept of area occupancy by independently assessing the inherent properties of heterogeneous, no lane-disciplined traffic streams. The authors have proposed the alternate measures to the three fundamental traffic flow characteristics incorporating the vehicle heterogeneity and the no lane-disciplined driving. While proposing this, it was ensured that the fundamental relations hold good.

The paper is organized into 5 sections. Section 2 provides a detailed discussion of the generalized definitions of the traffic stream characteristics proposed by Edie. Section 3 describes the proposed definitions of the traffic flow characteristics for the heterogeneous, no lane-disciplined traffic stream. In Section 4, the advantageous of the proposed definition over the existing definition are discussed. The proposed definitions are evaluated with the empirical data and presented in Section 5. Section 6 provides the conclusion of the study. In Section 7 the future scope of the study is discussed.

2. Edie's Generalized Definitions of the Traffic Flow Characteristics

Averaging the traffic flow characteristics was a fundamental problem to the traffic research community due to the ambiguity in the kind of averages to be employed. Depending on the measurement methods, various averaging techniques were used such as the arithematic, harmonic, spatial, or temporal averaging. All the averaging methods mentioned above are correct when used with the corresponding method of measurement. For example, when the vehicles are counted spatially for a number of instances (time-lapse photography), the temporal average will give the density. Similarly, when the traffic is counted at different road sections over a period of time, the spatial average will give the flow. Edie (1963) has proposed the generalized definitions for the flow characteristics which are independent of the measurement method and applicable to all kinds of measurements. These definitions unified the former definitions and eliminated the ambiguity about the method of averaging. Edie's generalized definitions of flow, density, and speed are widely accepted because of its ability to describe the fundamental relation as an identity. Edie (1963) have considered a time-space region of interest (as shown in Figure 1(a)), where the vehicle trajectories are present. Each vehicle trajectory is represented as a time space vector (x_i , t_i) where ' x_i ' is the distance travelled in the longitudinal direction and ' t_i ' is the time taken by the i^{th} vehicle (Figure 1(b)). The traffic flow characteristics such as the flow and the density are estimated by averaging the vectors together in space and time.

According to Edie (1963), the average flow is defined as;

$$Flow = \frac{Total \ distance \ travelled \ by \ all \ the \ vehicles}{Area \ of \ the \ 'time - space' \ region} \tag{1}$$

$$q = \frac{\sum_{i=1}^{n} x_i}{|A|} \tag{2}$$

and the density is defined as;

$$Density = \frac{Total time spent by all the vehicles}{Area of the 'time - space' region}$$
(3)

$$k = \frac{\sum_{i=1}^{n} t_i}{|A|} \tag{4}$$

$$u = \frac{q}{k} = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} t_i}$$
(5)



Fig. 1: Two dimensional representation of the vehicle trajectories and the time-space region



Fig. 2: Three dimensional representation of the vehicle trajectories and the time-space continuum

In his paper, Edie (1963) has stated that, "...While developed for one-dimensional flow of roadway vehicles, it could be extended to other types of vehicles and to additional dimensions of space". The authors have explored the possibility of applying these definitions for the heterogeneous, no lane-disciplined traffic stream. For capturing the vehicle heterogeneity and no lane-disciplined driving, the authors have incorporated an additional dimension of space to the existing definitions. The details of the proposed definitions are given in the following section.

3. Modified Generalized Definitions of the Traffic Flow Characteristics

The traffic stream in most of the developing countries is highly heterogeneous and no lane-disciplined. For an appropriate characterization of such traffic stream, it is important to identify certain characteristics which can capture both the heterogeneity and the no lane-disciplined driving actions. The above definitions (Eq. 2,4, and 5) are inadequate to represent the vehicle heterogeneity and no lane-disciplined driving actions. Past studies have showed that the analysis of heterogeneous streams with conventional definitions has achieved limited success and has involved much recalibration effort and a significant model modifications (Oketch, 2000). In an attempt to characterize the heterogeneous traffic stream, we have modified the existing definitions of the traffic flow characteristics. The new definitions incorporate an additional dimension of the space to the Edie's generalized definitions.

As suggested by Edie, the vehicle heterogeneity and the no lane-disciplined driving actions can be captured by considering the lateral dimension of the space. The lateral dimension of the road space could capture the no lanedisciplined driving, which is nothing but the frequent lateral movements. Similarly, the vehicle heterogeneity can be capture by considering the lateral dimension of the vehicles. By virtue of this, the two dimensional time space region in the Edie's generalized definition, hence transformed to a time space continuum with two dimensions of space and a time dimension, as shown in Figure 2(a). Each vehicle trajectory is then represented as a time space vector (x_i, y_i, t_i) where ' x_i ' is the longitudinal component of vehicle position, ' y_i ' is the lateral component of vehicle position, and ' t_i ' is the time taken by the *i*th vehicle (Figure 2(b)). The average traffic flow characteristics such as the flow and the density were estimated by averaging the vectors together in space and time. The concentration of the traffic stream is represented with a new parameter named as the "Area Density". The area density is defined as the fraction of time the three dimensional time-space continuum is occupied by the vehicles. Mathematically it is expressed as,

$$Area Density = \frac{Sum of the areas of the time space region occupied by each vehicle}{Volume of the 'time - space' continuum}$$
(6)

$$k_A = \frac{\sum\limits_{i=1}^{i} t_i \times w_i}{L \times W \times T}$$
(7)

Similarly, the throughput is termed as "Area Flow", and is defined as Proportion of the road space utilized by all the vehicles within the time-space continuum during the observation period. And is expressed as,

$$Area Flow = \frac{Sum of the areas of the projected path of each vehicle}{Volume of the 'time - space' continuum}$$
(8)

$$q_A = \frac{\sum\limits_{i=1}^{n} d_i \times w_i}{L \times W \times T}$$
(9)

The ratio of the Equations 9, and 8 is the rate at which the road space is freed up. i.e.,

$$RFR = \frac{q_A}{\rho_A} = \frac{\sum d_i \times w_i}{\sum t_i \times w_i}$$
(10)

This parameter, named as Road Space Freeing Rate (RFR) is similar to the average traffic stream speed in the homogeneous, lane-disciplined traffic stream. In case of heterogeneous traffic stream, it may not be appropriate to consider the ratio of area flow and area density as the average stream speed since the speed characteristics among the vehicle classes varies significantly. Similar to the speed and density, RFR is inversely proportional to the area density, i.e., higher/lower the area density, lower/higher be the RFR.

4. Advantages of the Proposed Definitions over the Edie's Definitions under Heterogeneous, No Lane-Disciplined Traffic Conditions

The advantages of the proposed definitions over the generalized definitions have been highlighted by applying both the definitions on two hypothetical traffic streams. The first stream has only cars whereas the second stream has only motorized two wheelers (MTW) and both the streams are no lane-disciplined. Let t_i , (where, i = 1, 2, ..., N), be the time taken by i^{th} car or i^{th} MTW to cross a given road length *L*. Let the number of cars and bikes crossing the road during the observation period be the same (*N*) and no vehicles were found on the road stretch at the starting and ending of the observation period. The crowdedness in the car stream and the MTW stream has to be different due to the difference in the vehicle dimensions. Now, let us compare the densities of both the car and MTW streams that is estimated using Edie's definition and the modified definition.

Edie's Definition:

The traffic concentration corresponding to the car traffic,

$$k_c = \frac{\sum_{i=1}^N t_i}{|A|} \tag{11}$$

The traffic concentration corresponding to the bike traffic,

$$k_{MTW} = \frac{\sum_{i=1}^{N} t_i}{|A|} \tag{12}$$

from Equation 11 and 12,

$$k_{MTW} = k_c \tag{13}$$

As discussed earlier, the Equation 13 is not true because of the difference in the vehicle dimensions.

Modified Definition:

n

The traffic concentration corresponding to the car traffic,

$$k_c = \frac{\sum\limits_{i=1}^{c} t_i \times w_{c_i}}{L \times W \times T}$$
(14)

The traffic concentration corresponding to the bike traffic,

$$k_{MTW} = \frac{\sum\limits_{i=1}^{n} t_i \times w_{MTW_i}}{L \times W \times T}$$
(15)

As $w_{c_i} \neq w_{MTW_i}$,

 $k_{MTW} \neq k_c$

It is evident from the above analysis that by using the Edie's generalized definition, both the assumed traffic states result in a same density value as the times spent are equal. Whereas the proposed definitions will take care of the vehicle dimensions and hence will give a more meaningful representation of the traffic states.

On the other hand, it is convenient to assume the homogeneous, lane-disciplined traffic stream is a special case of the heterogeneous, no lane-disciplined traffic stream. Hence, the proposed definitions could be deduced to the Edie's generalized definitions when considered the lane-discipline and the vehicle homogeneity. For homogeneous traffic stream it is convenient to assume the equal physical dimensions for all the vehicles as $(l \times w)$, where *l* is the length and *w* is the width of the vehicle. As all the vehicles are of same width *w*, the expression for flow (Equation 9) will become,

$$q_A = \frac{w}{W} \times \frac{\sum_{i=1}^{n} d_i}{L \times T}$$
(17)

The one dimensional analysis of the homogeneous and lane-disciplined traffic stream allow to neglect the terms corresponding to the second dimension (w/W) and to consider that in the unit part. It is clear from the above expression

that the term w represents the vehicle type, W stand for the road width for which the flow is defined, and $\sum_{i=1}^{n} d_i/L$ counts

the number of vehicles found during the observation period T. Looking into the units, w can be given with an objectspecific unit that is the vehicle, and the W represents the lane (as the traffic is lane-disciplined, flow can be represented in 'per lane' unit which is the scaled unit of 'per unit road width'). With all the above considerations, the Equation 17 can be deduced to,

$$q_A = \frac{\sum\limits_{i=1}^{n} d_i}{L \times T} Veh/Hour/Lane$$
(18)

which is nothing but the Edie's generalized definition for flow. Similarly, the density,

$$k_A = \frac{w}{W} \times \frac{\sum_{i=1}^{n} t_i}{L \times T}$$
(19)

where the $\sum_{i=1}^{n} t_i/T$ counts the vehicles over the study road stretch (*L*).

$$k_A = \frac{\sum\limits_{i=1}^{n} t_i}{L \times T} Veh/km/Lane$$
(20)

The RFR for homogeneous traffic stream is nothing but the average stream speed due to the similar speed characteristics among the vehicles. The above analyses have proved that the proposed definitions are generic to any traffic stream configurations such as the lane-disciplined and/or no lane-disciplined, homogeneous and/or heterogeneous.

5. Empirical Investigation of the Utility of the Proposed Definitions

Trajectories of the vehicles moving over a 60-meter urban mid-block section at Guwahati city, India, were extracted using the Traffic Data Extractor (TDE, Munigety et al. (2014)). The extracted trajectories were reconstructed using the methodology proposed by Suvin and Mallikarjuna (2018). Both the Edie's generalized definition and the Modified definition were applied to estimate the flow, density, and speed at a time aggregation of 1-minute. The outliers in the flow values were removed after quartile analysis of the binned density data (Bin Size = 20 Traffic unit/Km/Lane). After removing the outliers, the flow and density were plotted along with the speed contour (Figure 3). It is evident from the figure that the modified definitions provide more consistent variation of the traffic states. For example, corresponding to the flow of 1750 Traffic unit/Hr/Lane (Figure 3(a)), the speed variation with the increase in density is not proportional. Where as, the modified definition shows a comparatively commensurable variation of the speed with density and flow (Figure 3(b)). The flow-density data estimated using both the generalized and the modified



Fig. 3: Performance Analysis of the Proposed Definitions

generalized definition were fitted with three regime piecewise linear model as shown in Figure 3(c). It can be observed from the figure that the capacity-flow is significantly lower when estimated using the generalized definition compared to that with the modified definitions. Also, it is evident from the figure that the traffic breakdown occurring at a lower density values when the traffic is characterized with the Edie's generalized definition, which is not the case in the field. It is a known fact that the seeping or filtering behaviour, side-by-side movements of vehicles, and so on enhance the capacity of the roadway to a certain extent. The modified definitions could capture such behaviour through the added additional dimension of the space and it is clearly evident from Figure 3(c).

6. Summary and Conclusion

The present study has proposed a set of new definitions for the traffic stream characteristics corresponding to the heterogeneous, no lane-disciplined traffic stream. Edie's generalized definitions of the traffic stream, which are capable of describing the fundamental relationship as an identity, extended to heterogeneous and no lane-disciplined traffic conditions. It was found that the vehicle heterogeneity and the no lane-disciplined driving behaviour are attributed to the lateral dimension of the vehicle type and the road space, respectively. In order to capture the heterogeneity and the no lane-disciplined driving actions, the lateral dimension of the space is introduced into the Edie's definitions. The modified definitions consider two dimensions of the space (Longitudinal and Lateral) and a time dimension. The vehicle dynamics are represented in the three dimensional time-space continuum, and the traffic stream characteristics within the continuum are expressed by time spent and the distance travelled by all the vehicles in the continuum. The time spent by the vehicles measures the crowdedness of traffic and represented with '*area density*'. Whereas, the distance travelled is a measure for the traffic intensity which is represented with '*area flow*'. The ratio between the area flow and the area density is considered as the rate at which the road space is emptied (RFR).

Advantages of the proposed definitions over the existing definitions are examined. It was found that, compared to the Edie's definitions, the proposed definitions are able to represent the heterogeneous and no lane-disciplined traffic stream in a more sensible way. The consideration of the lateral dimension of the vehicles captures the vehicle heterogeneity the incorporation of the road width captures the no lane-disciplined driving. This study has proved that the proposed definitions are more generic and can be applied to any traffic stream configuration.

7. Future Scope

The definitions of the traffic flow characteristics proposed herein suggest a new way of looking at the heterogeneous, no lane-disciplined traffic stream. This paper has solved only a small part of the complexity associated with the heterogeneous, no lane-disciplined traffic stream. Along with the vehicle heterogeneity and the no lane-disciplined driving behaviour, the friction experienced by the vehicles from the road side activities also contribute to the traffic stream behaviour. The side friction experienced by the moving traffic stream can be a consideration for the future study.

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